Optimal control of biogas production in anaerobic digestion bioreactors

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Anaerobic Digestion Models

- ADM1 : 19 biochemical processes, 7 populations, 26 state variables
- Bernard 2001 : 2 stage model
  - S_1 : organic substrate, S_2 : VFA
  - X_1 : acidogenic bacteria, X_2 : methanogenic bacteria,
  - μ_1 : Monod kinetics, μ_2 : Haldane kinetics.

$$\dot{S}_1 = D(S_{in,1} - S_1) - k_1\mu_1(S_1)X_1$$
$$\dot{X}_1 = \mu_1(S_1)X_1 - \alpha DX_1$$
$$\dot{S}_2 = D(S_{in,2} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2$$
$$\dot{X}_1 = \mu_2(S_2)X_2 - \alpha DX_2$$

- Methane outflow :

$$Q_{CH_4} = k_M\mu_2(S_2)X_2$$

AD model : 1 stage model

Simplified model for control purposes :

- A single bioprocess : Methanogenesis

$$S \to X + CH_4$$

- Continuous flow stirred-tank reactor (CSTR) : Chemostat model

$$\begin{aligned} \text{Substrate} \quad & \dot{S} = D(S_{in} - S) - \mu(S)X \\ \text{Biomass} \quad & \dot{X} = \mu(S)X - DX \end{aligned}$$

- Methane outflow

$$Q_{CH_4} = \mu(S)X$$

- Optimal control problem :

$$\max_{D} \quad \int_{t_0}^{T} \mu(S(t))X(t)dt$$

Optimal control for a specific class of initial conditions

1 stage model reduced to 1 equation with $s_{in} = s_0 + x_0$:

$$\dot{s} = \gamma(s) - D(s_{in} - s)$$

with $\gamma(s) = \mu(s)(s_{in} - s)$. The optimal control problem is :

$$\max_D \int_0^T \gamma(s(t))dt \qquad \text{with } D \in [D_{min}, D_{max}]$$

and has been solved :

$$D(t) = \begin{cases} D_{min} & \text{if } s(t) > s_* \\ D_{max} & \text{if } s(t) < s_* \\ D_* & \text{if } s(t) = s_* \end{cases}$$

where $s_* = \text{argmax } \gamma(s)$ and $D_* = \mu(s_*)$ is the dilution rate for which s_* is a steady state

General initial conditions

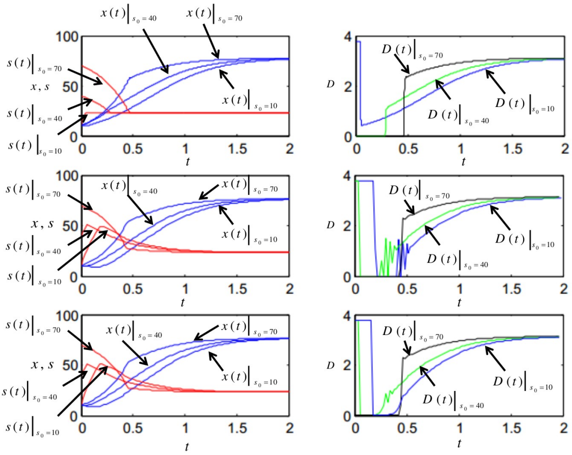
- We can adapt this control to the full 2 equation model :

$$D(t) = \begin{cases} D_{min} & \text{if } s(t) > s_* \\ D_{max} & \text{if } s(t) < s_* \\ D_* \frac{x(t)}{s_{in} - s_*} & \text{if } s(t) = s_* \end{cases}$$

where $s_* = \text{argmax } \mu(s)(s_{in} - s)$ and $D_* = \mu(s_*)$

- We conjecture that this the optimal control after a numerical study of the problem

BOCOP computations



from “Analyse et controle optimal d’un bioreacteur de digestion anaerobie”, PhD thesis of Amel Ghouali, 2015

Hamilton Jacobi Bellman

Value function with $\xi = (S(t_0), X(t_0))$:

$$V(t_0, \xi) = \max_D \int_{t_0}^{T} \mu(S(t))X(t)dt$$

Hamilton Jacobi Bellman equation :

$$\partial_{t_0} V(t_0, \xi) + H(\xi, -\nabla_\xi V(t_0, \xi)) = 0 \tag{HJB}$$

Hamiltonian with f the dynamics :

$$H(X, S, p) = \sup_D \left\{ \mu(S)X - p \cdot f(X, S) \right\}$$

We want to show that the cost function associated to the proposed control verifies (HJB)

In progress

We start with $s_0 > s_*$. In this case we have :

$$s(t, t_0, \xi, D^*) = \begin{cases} s(t, t_0, \xi, D_{\min}) & \text{for } t_0 \leqslant t \leqslant t_* \\ s_* & \text{for } t_* < t \leqslant T \end{cases} \quad (1)$$

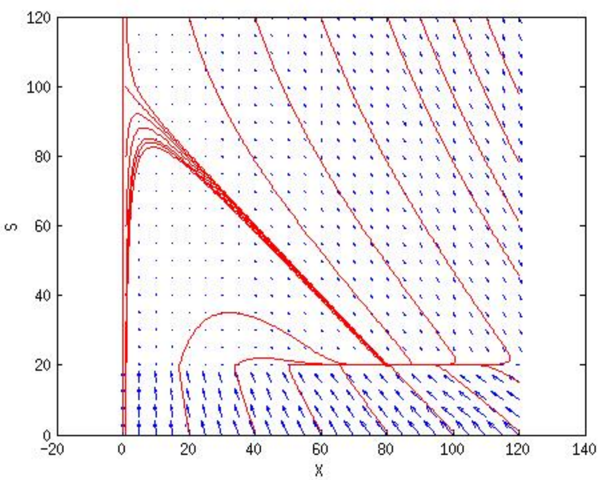
and for $z = x + s$

$$z(t, t_0, \xi, D^*) = \begin{cases} s_{in} + (z_0 - s_{in}) e^{-D_{\min}(t - t_0)} & \text{for } t_0 \leqslant t \leqslant t_* \\ s_{in} + (z_0 - s_{in}) e^{-D_{\min}(t_* - t_0) - D_*(t - t_*)} & \text{for } t_* < t \leqslant T \end{cases} \quad (2)$$

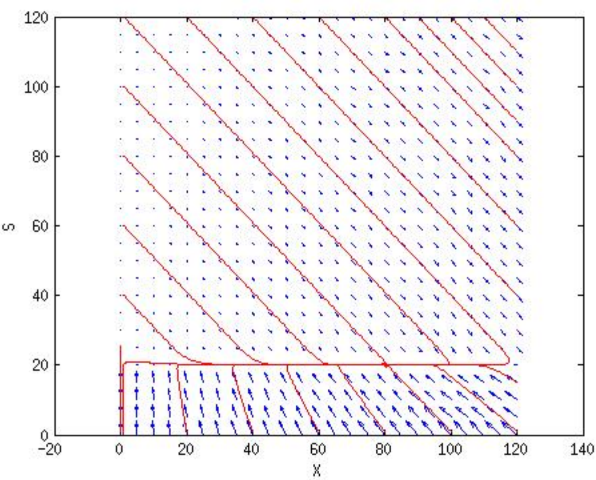
and we compute the partial derivatives of the associated cost function and show that they verify (HJB)
but for now :

$$(HJB) = \mu(s_*)(z(T) - z(t_*))(\frac{D_{\min}}{D_*} - 1)$$

1 stage model : vector field, ($D_{min} > 0$)

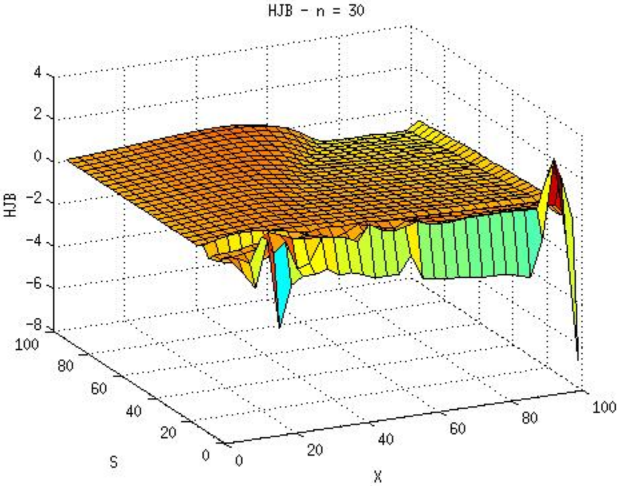


1 pop model : vector field ($D_{min} = 0$)



Numerical verification of HJB

- Simulations on a grid of initial conditions → Cost function
- Numerical derivatives → Hamiltonian



In progress and future work

Numerical resolution of (HJB) with ROC-HJ software

- Boundary conditions?
- State constraints : $X, S \geqslant 0$
- Reconstruction of trajectories using the Dynamic Programing Principle

More complex models

- Models for solid waste treatement
- 2+ stage models : numerical vs analytic optimal control
- PDE models
  - Simulations of AD bioreactor and 'virtual' validation of control strategies from ODE models
  - Optimal design and influence of geometry