Algorithmic Game Theory

Auction Games, I

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Outline

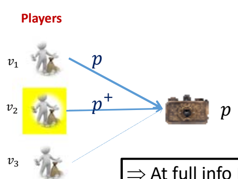
- Monday: games, Price of anarchy, smoothness based proof in congestion games
- Tuesday: learning as a behavior in games (instead of finding Nash)
- No-regret learning converges to coarse correlated equilibria

  in T steps regret error O($\sqrt{T \ln n}$)
  - Hence coarse correlated equilibria computable
  - Note, it's a linear program (in exponential number of variables):
    - Variables p(s) probability of strategy vector s
    - Constraints: no regret for each player i and each strategy $s_i' \in S_i$

Today: Auctions as games

First Example: Single item first price

- Auction sets a price p (full info, pure Nash).

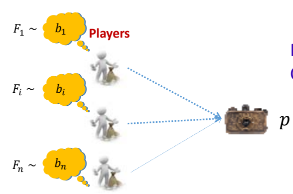


Can win by bidding p⁺

⇒ At full info Nash player with max v_i wins

First price auction with uncertainty?

- Bayesian game
- Randomized bid



Price p random

Cannot bid p⁺

Today utility games:

- Finite set of players 1,…,n
- strategy sets S_i for player i: bid on some items

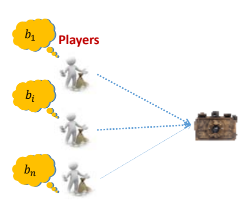
  not a finite set
- Resulting in strategy vector: s=$(s_1, \dots, s_n)$ for each $s_i \in S_i$
- Utility player i: $u_i(s)$ or $u_i(s_i, s_{-i})$
  - We assume quasi-linear utility, and no externalities:
  - If player wins set if items A_i and pays p_i her value is $v_i(A_i) - p_i$
- Pure Nash equilibrium if $u_i(s) \geq u_i(s_i', s_{-i})$ for all players and all alternate strategies $s'_i \in S_i$

Bayes Nash analysis

Strategy: bid as a function of value $b_i(v)$

Nash: $E_{v_{-i}b}[u_i(b(v))|v_i] \geq E_{v_{-i}b_{-i}}[u_i(b_i', b_{-i}(v_{-i}))|v_i]$

for all b_i'



Example: [0,1] uniform value independent

- Two players:
- Assume both use deterministic, monotone, and identical bidding functions b(v)
  - Person with larger value wins
  - Bid must maximize utility:
    alternate bid for a player with value v: bid b(z) (pretend to have value z)

$v = \text{argmax}_z\, z\,(v - b(z))$ → v-b(v)-vb'(v)=0

Prob of winning   value   price

Solved by b(v)=v/2

First price single item auction

- Uniform independent [0,1] value n players:
  bid b(v)=$\frac{n-1}{n}v$ (more competition bid more aggressively)
- Independent identical distributions $\mathcal{F}$ and n players:
  bid b(v)= E(max of n-1 draws from $\mathcal{F}$|each $\leq v$)

BTW, Second price auction: bid your value,
first price bid = expected payment
revenue equivalence (Meyerson)
If distribution not identical and independent: big mess!!!

Smoothness for auctions

Auction game is (λ,μ)-smooth if for some μ>1, λ>0 and some strategy s* and all s we have

$$\sum_i u_i(s_i^*, s_{-i}) \geq \lambda opt - \mu\, R(s)$$

R(s) = revenue at bid vector s (usually μ=1)

Theorem: [Syrgkanis-T'13] Price of anarchy for any (λ,μ)-auction game is at most $\mu\,/\lambda$

Social welfare: $\sum_i u_i(s) + R(s)$

Smoothness for utility games

Regular game is (λ,μ)-smooth if for some μ<1, λ>0 and some optimal strategy s* all s we have

$$\sum_i c_i(s_i^*, s_{-i}) \geq \lambda c(s^*) + \mu c(s)$$

Utility game is (λ,μ)-smooth if for some μ, λ>0 and some optimal strategy s* all s we have

$$\sum_i u_i(s_i^*, s_{-i}) \geq \lambda u(s^*) - \mu u(s)$$

Theorem: [Roughgarden'09] Price of anarchy at most $(1+\mu)/\,\lambda$

Smoothness for auctions

Utility game is (λ,μ)-smooth if for some μ, λ>0 and some optimal strategy s* and all s we have

$$\sum_i u_i(s_i^*, s_{-i}) \geq \lambda u(s^*) - \mu u(s)$$

Auction: extra player with no strategies: auctioneer. Value p(s)=price
Social welfare: $\sum_i u_i(s) + R(s)$

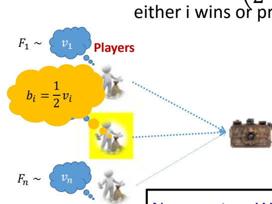
Auction game is (λ,μ)-smooth if for some μ>1, λ>0 and some strategy s* all s we have

$$\sum_i u_i(s_i^*, s_{-i}) \geq \lambda opt - \mu\, R(s)$$

Robust Analysis: first price auction

No regret: $u_i(b) \geq \quad u_i\left(\frac{1}{2}v_i, b_{-i}\right) \geq \frac{1}{2}v_i - p$

either i wins or price above p $\geq \frac{1}{2}v_i$



Bid b_i=$\frac{1}{2}v_i$
No need to bid $\frac{1}{2}v_i$ …
Just don't regret this!

No regret ⇒ Winner has value $\geq \frac{1}{2}v_i$ ∀i
Price of anarchy ≤ 2 (1.59…)

Homework problem setup

- Auction game with players having value functions $v_i(k)$ as a function of items won. Assume $v_i(k)$ is concave.
- You have K items to sell
- Auction A: sort request by marginal value, assign to the top K bidders charging the last allocated items claimed value as price to all
- Auction B: sort request by marginal value, assign to the top K bidders charging the first unallocated items claimed value as price to all

Example $v_1(k)$=2k, and $v_2(k) = 5\sqrt{k}$ and K=3

Marginal values $v_2(1) = 5$, $v_2(2) - v_2(1) = 5(\sqrt{2} - 1) \approx 2.07$, $v_1(1) = 2$, $v_1(2) - v_1(1) = 2$

Both auctions give 1 to player 1 and 2 to player 2, and charge 2 for all items

Questions

- Is Auction A truthful (that is, is bidding your two value always a good strategy)
- Is Auction B truthful?
- Challenge: Show that Auction A is (1/2, 1)-smooth (and hence has a price of anarchy of at most 2