School on Information and Randomness
Center for Mathematical Modeling, University of Chile
December 5–9, 2016, Santiago-Chile

Ellen Baake

University of Bielefeld, Germany

Title: *Solving the recombination equation*

Abstract: The recombination equation is a well-known dynamical system from mathematical population genetics, which describes the evolution of the genetic composition of a population that evolves under recombination. The genetic composition is described via a probability distribution (or measure) on a space of sequences of finite length; and recombination is the genetic mechanism in which two parent individuals are involved in creating the mixed sequence of their offspring during sexual reproduction. The model comes in a continuous-time and a discrete-time version; it can accommodate a variety of different mechanisms by which the genetic material of the offspring is partitioned across its parents. In all cases, the resulting equations are nonlinear and notoriously difficult to solve. Elucidating the underlying structure and finding solutions has been a challenge to theoretical population geneticists for nearly a century. In this talk, we show how this equation can be solved, in two ways: forward in time, via a modern version of so-called Haldane linearisation; and backward in time via an associated stochastic fragmentation process. This is joint work with Michael Baake. *Reference:* E. Baake, M. Baake, Haldane linearisation done right: Solving the recombination equation the easy way, Discr. Cont. Dyn. Syst., in press.

Michael Baake

University of Bielefeld, Germany

Title: *Dynamical systems of number-theoretic origin in the theory of aperiodic order*

Abstract: Cut and project sets, which go back to Yves Meyer (1972) in mathematics and to Peter Kramer (1982) in physics, are a versatile class of structures with amazing harmonic properties. These sets are also known as mathematical quasicrystals, and include the famous Penrose tiling with fivefold symmetry as well as its various generalisations to other non-crystallographic symmetries. These constructions are widely used to model the structures discovered in 1982 by Dan Shechtman (2011 Nobel Laureate in Chemistry). More recently, also systems such as the square-free integers or the visible lattice points have been studied in this context, leading to the theory of weak model sets. This talk will review some of the developments, and introduce important concepts of the field, with focus on spectral aspects.

School on Information and Randomness
Center for Mathematical Modeling, University of Chile
December 5–9, 2016, Santiago-Chile

Pierre Collet

CNRS, École Polytechnique, France

Title: *Quasi stationary distributions for large population birth and death processes. Can we see it? What can we do with it?*

Abstract: We consider a birth and death process (mono type or multi type) with a parameter K (carrying capacity) related to the scale of the population size. For large K we prove there exists a unique quasi stationary distribution (qsd). The time scale of convergence to the qsd statistical regime is much smaller than the time scale of extinction of the population, therefore this regime is observed. In a mono type system the linear resilience can be expressed in terms of the average jump rate and variance of the population in the qsd.

Fernando Cordero

University of Bielefeld, Germany

Title: *Lines of descent in a deterministic model of mutation and frequency dependent selection*

Abstract: We consider the diploid mutation-selection equation that arises in the limit of large populations (without any rescaling of parameters or time) of the two-type Moran model with mutation and frequency dependent selection. While the forward in time behaviour of this model is well understood, the ancestral picture is still missing. To fill in this gap, we consider an asymptotic version of the ancestral selection graph (ASG) with pairwise interaction. The lines in the ASG represent potential ancestors of a sample of the population, or individuals which have interacted with them. The evolution of the asymptotic ASG includes bifurcations and trifurcations. Each trifurcation event create two new families of lines, whereas bifurcation events only increase in one the number of lines in a given family. We associate to each family of lines a natural hierarchy. In addition, we develop a pruning procedure which takes into account the effect of the mutation events. Based on this ideas, we construct a Markov process with values on the set of ternary trees with marked leaves, which is in duality with the forward process. The corresponding duality function is defined in a recursive way. Finally, we construct a new Markov process with values on the set of labelled directed graphs. The latter is also in duality with the forward process, this time through an explicit duality function.

Sebastián Donoso

Universidad de Chile, Chile

Title: *Topological structures and the pointwise convergence of some averages for commuting transformations*

Abstract: In this talk I will discuss some recent results in the pointwise convergence of some averages for commuting transformations. I will focus on the pointwise convergence of multiple averages for distal commuting transformations. I will show the main ideas we use, which combine the notions of sated extensions by T. Austin, magic extensions by B. Host and ideas by W. Huang, S. Shao an X. Ye on introducing a topological structure to build a strictly ergodic model for an ergodic system. This is a joint work with Wenbo Sun.

Pablo Ferrari

Universidad de Buenos Aires, Argentina

Title: *Ergodic invariant measures for the Ball Box System*

Abstract: The BBS is a cellular automaton introduced by Takahashi and Satsuma in 1990 as a discrete analog of the KdV differential equation which has many soliton solutions.

A carrier with infinite capacity travels from left to right along boxes located at the integers. Each box may contain one ball or be empty. The carrier picks balls from occupied boxes and leaves carried balls at empty sites. If the initial configuration of balls has density less than $1/2$, the automaton is well defined in Z. The product measure at any density less than $1/2$ is invariant. The automaton has many conserved quantities and (non-product) invariant measures. We describe the set of all spatially ergodic invariant measures for the BBS. Work in collaboration with Chi Nguyen, Leonardo Rolla and Minmin Wang.

Damien Gaboriau

Center for Mathematical Modeling and École Normale Sup. Lyon, Chile and France

Title: *Approximations for products in orbit equivalence*

Abstract: Hyperfiniteness consists in approximation of the orbits of a group ac- tion by finite pieces. It plays a central role in orbit equivalence. In a joint work with Robin Tucker-Drob we initiate a general study of the notion of approximation of probablility measure preserving actions by subrelations. For non-amenable product groups, we exhibit circumstances in which there exists no approximation at all. This result has a consequence in Bernoulli

percolation on Cayley graphs of such groups: the uniqueness thresh- old pu doesnâĂŹt belong to the uniqueness phase.

Antonio Galves

Universidad de Sao Paulo, Brazil

Title: *EEG, the goalkeeper game and the stochastic brain hypothesis: comparing a driving stochastic chain and a blurred version*

Abstract: We consider two stochastic chains naturally coupled in such a way that given the past they have a small probability of differing. This situation presumably occurs in brain activity when a chain of stimuli is presented to a volunteer and we observe a corresponding chain of neurophysiological recordings. The question is how these two chains are quantitatively related. In this talk I will recent results both mathematical and neurobiological recently obtained by the NeuroMat team.

Felipe García Ramos

Universidad Autónoma de San Luís Potosí, Mexico

Title: *Mean proximality and mean Li-Yorke chaos*

Abstract: Using averages one can define mean versions of classical topological dynamics notions. These have a closer relationship with ergodic theory. In this talk we will characterize mean proximal systems, and we will show how to use mean proximality to obtain mean Li-Yorke chaotic systems (also known as DC2 chaos).

Yonatan Gutman

Polish Academy of Sciences, Poland

Title: *Higher order regionally proximal equivalence relations for general group actions*

Abstract: Ellis and Gottschalk introduced the regionally proximal relation in 1960 whereas the higher order regionally proximal relations for abelian group actions were introduced by Host, Kra and Maass in 2010. The interest in these relations lies in their role in the structural theory of topological dynamical systems. For amenable minimal group actions the regionally proximal relation

is an equivalence relation but for some non-amenable minimal actions it is not. When it is an equivalence relation, the quotient by the regionally proximal relation is the maximal equicontinuous factor. In this talk I will introduce a generalization of the higher order regionally proximal relations suitable for an arbitrary acting group. The surprising new main result is that these generalized relations are always equivalence relations for an arbitrary minimally acting group. Moreover the generalized regionally proximal equivalence relation of order one corresponds to the maximal (compact) abelian group factor, yielding for the first time an explicit description of this factor in the category of non-abelian minimal actions. Joint work with Eli Glasner and XiangDong Ye.

Thierry Huillet

CNRS, University of Cergy-Pontoise, France

Title: *Variations on the Luria and Delbrück model.*

Abstract: One of the most popular models for quantitatively understanding the emergence of drug resistance both in bacterial colonies and in malignant tumors was introduced in 1943 by Luria and Delbrück. Here, individual resistant mutants emerge randomly at birth events embedded in an exponentially growing sensitive population. The Luria-Delbrück experiment (known as the Fluctuation Test) demonstrates that genetic mutations of bacteria arise permanently, even in the absence of selection, rather than being a response to selection thereby justifying the latter scenario. It was thus confirmed that mutations do not occur out of necessity (a Lamarckian approach), but instead can occur many generations before the selection strikes (the Darwinian point of view). We shall unravel some of the probabilistic aspects of this problem together with some of its variations, including the opportunity of a linearly growing sensitive population.

Sylvie Méléard

École Polytechnique, France

Title: *Stochastic dynamics for adaptation and evolution of microorganisms*

Abstract: Understanding the adaptation and evolution of populations is a huge challenge, in particular for microorganisms since it plays a main role in the virulence evolution or in bacterial antibiotics resistances. We propose a general eco-evolutionary stochastic model of population dynamics with clonal reproduction and mutations. The individuals compete for resources and exchange genes, as in the transfer of plasmids in bacteria. A large population

approximation leads to a nontrivial dynamical system. Stability is studied to analyze the conditions for the fixation of a mutant or the maintenance of polymorphism. In the first case, we also provide the probability of fixation and time to fixation. Under the biological assumptions of the adaptive dynamics (large population and rare mutations), we show that at the (long) mutation time scale, the stochastic measure-valued population process converges to a jump process describing the successive invasions of successful mutants. When restricted to the case without gene transfer, this process is usually known as Trait Substitution Sequence (see Metz et al. 1996). We explain how the gene transfer can drastically affect the evolutionary outcomes. Simulations show its effect on the elimination of pathogens strains but also the appearance of resistance patterns in very long times. This work is developed with S. Billiard, P. Collet, R. Ferrière and C.V. Tran.

Karl Petersen

University of North Carolina, USA

Title: *Average sample complexity*

Abstract: The complexity function of a sequence or subshift counts the number of words of each length, and topological entropy is its exponential growth rate. In joint work with Benjamin Wilson we consider the number of words that can be seen in a finite sampling window, not necessarily an interval, among all sequences in a subshift. We define the average sample complexity to be the exponential growth rate of the average of this quantity over all windows within an interval of fixed length. The measure-theoretic version takes into consideration the probabilities of the observed patterns. There are also extensions to pressure. The idea arises from a dynamical analogue of the neural intricacy proposed by neuroscientists G. Edelman, O. Sporns, and G. Tononi and studied in the probabilistic setting by J. Buzzi and L. Zambotti. Intricacy measures the balance between freedom and interdependence; it is high when there are both local freedom and global organization, low when there is either complete freedom or complete order. We identify average sample complexity as fiber entropy in a skew product system. An interesting problem is to find the measures of maximal average sample complexity or intricacy. We show that subshifts have ergodic measures that maximize average sample complexity, but probably these maximizing measures are not Markov of any order and are supported on small subshifts.

Pierre Picco

CNRS, Institut de Mathématiques de Marseille, France

Title: *Phase Separation for the long range one-dimensional Ising model*

Abstract: We consider the problem of separation of phase for the one dimensional Ising model with long range interactions. We prove that in the regime of phase transition, conditionally to have an empirical magnetisation which is in between the two equilibrium spontaneous magnetisations $]m\beta, m\beta[$, typical configurations can be roughly speaking described as a macroscopic interval of the minus phase immersed in the plus phase.

Pablo Shmerkin

University Torcuato di Tella, Argentine

Title: *Furstenberg's conjecture on intersections of Cantor sets, and self-similar measures.*

Abstract: In 1970, H. Furstenberg proposed a conjecture on the dimension of the intersections of $\times 2$ and $\times 3$-invariant sets (and their affine images). I will outline the main tools involved in the recent solution of this conjecture, and discuss other results that can be obtained with the same techniques.

Michael Schraudner

Center for Mathematical Modeling, Chile

Title: *Automorphism groups of countable subshifts*

Abstract: Recent years have seen much activity in studying automorphism groups of "small" subshifts. Particular cases are various results on one-dimensional subshifts of low (i.e. linear, quadratic, polynomial) complexity, under the assumption of the subshift being minimal. The overall goal has been to describe the automorphism group Aut(X) as an abstract group, to investigate its subgroup structure and algebraic properties or to prove results about its growth, i.e. being or not amenable. In the talk we present a new technique of seeing Aut(X) as a group extension of simpler constituent groups and we apply this construction to explicitly compute and describe Aut(X) for several families of countable (sofic) Z-subshifts X. Our technique can be used to characterize, among other properties, residual finiteness of Aut(X) for countable Z-sofics as well as for some other classes of Z-subshifts. Finally we are able to prove amenability of the automorphism group for all countable subshifts on amenable groups, contrasting known results even in the case of one-dimensional subshifts (finite type, sofic) of positive entropy, whose automorphism groups have been

shown to be non-amenable and extremely complicated. (Joint work with V. Salo.)

Edgardo Ugalde

Universidad Autónoma de San Luís Potosí

Title: *Projective convergence of measures*

Abstract: We recently introduce a distance for processes, which is based on Hilbert's projective distance. This distance is strictly stronger than the $*$-weak distance, but it is not comparable with Ornstein's d-bar distance. We prove that all g-measure is the projective limit of its Markovian approximations and that the ergodic properties of the limit g-measure depend on the speed of convergence of these Markovian approximations. We also study the stationary distribution of constant length random substitutions and prove they are g-measures. I will finish the talk by formulating some problems we are currently working on. This is a work in collaboration with Liliana Trejo and César Maldonado, and is based on previous works in collaboration with Leticia Ramírez and Jean-Rene Chazottes.

Denis Villemonais

University of Lorraine, France

Title: *Uniform exponential convergence of conditioned processes*

Abstract: The aim of this talk is to recall the notion of quasi-stationary distribution and to present a necessary and sufficient criterion for the uniform exponential convergence of conditioned Markov processes to a unique quasi-stationary distribution. While this general criterion can be difficult to check in practical cases, we present new Lyapunov-type criteria which simplify the study of conditioned processes in many situations, including multi-dimensional birth and death processes and diffusion processes conditioned not to vanish. These results have all been obtained in collaboration with Nicolas Champagnat.