Two-scale stochastic dynamic optimization

application to energy storage in subway stations

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Optimization for subway stations

Subway stations consumption $=$ 40.000 houses

Energy transition of cities requires significant investments

Is electrical storage affordable for stations?

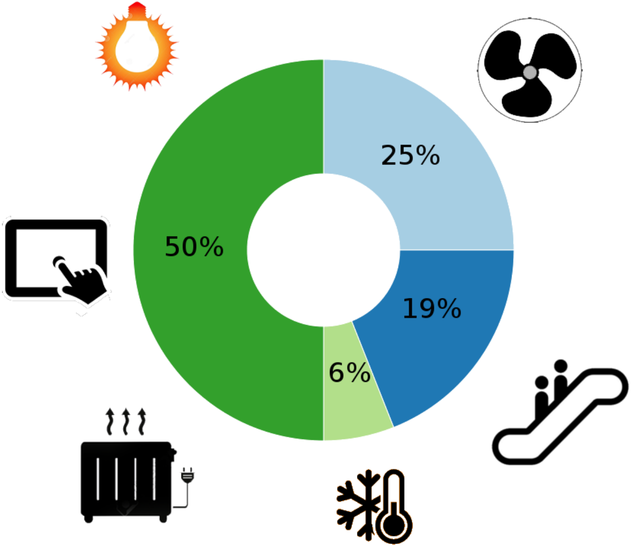
We use stochastic optimization for short term control and long term management of batteries

Outline

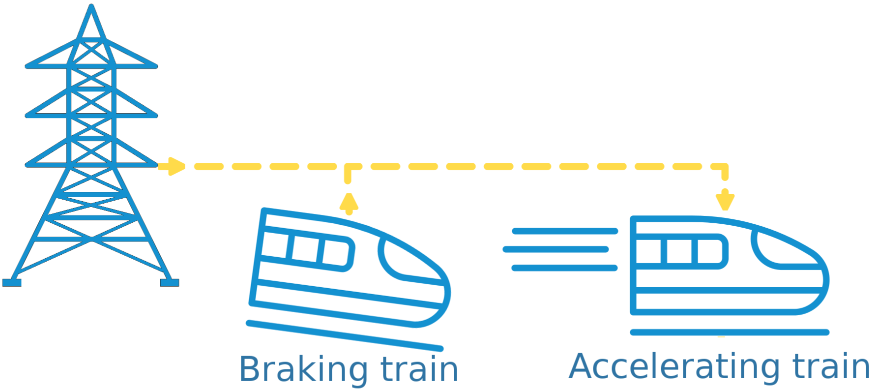
Outline

Why electrical storage in stations?

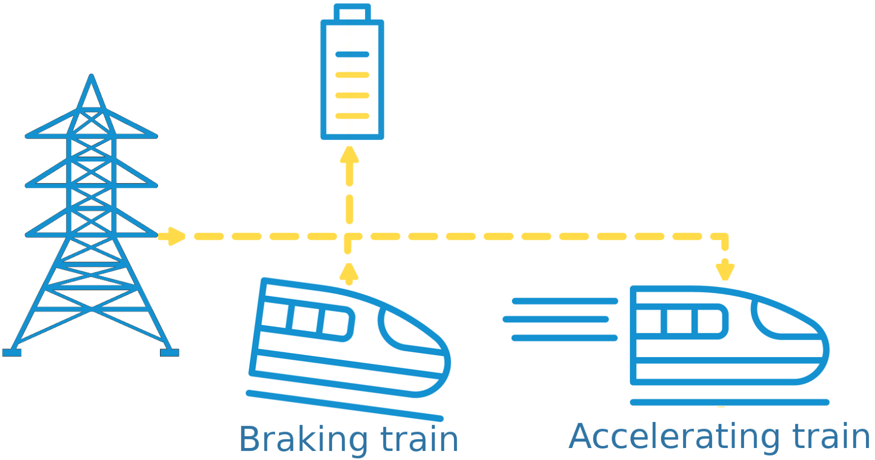
Subway stations typical energy consumption



Subway stations have unexploited energy ressources

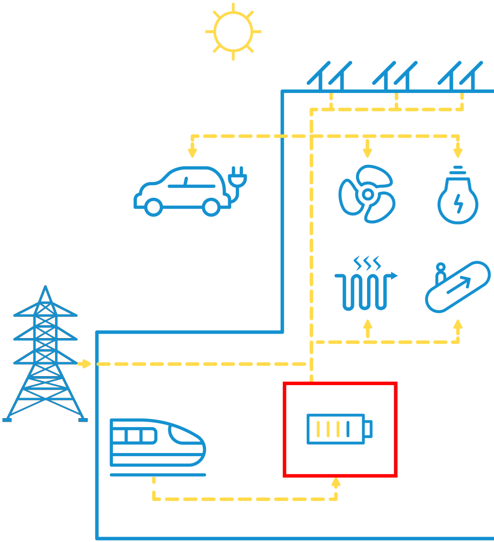


Energy recovery requires a buffer



Managing storage short term operations

Microgrid concept for subway stations



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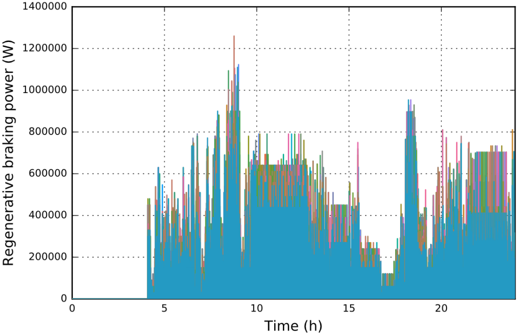
We need stochastic optimization

Subways braking energy is unpredictible



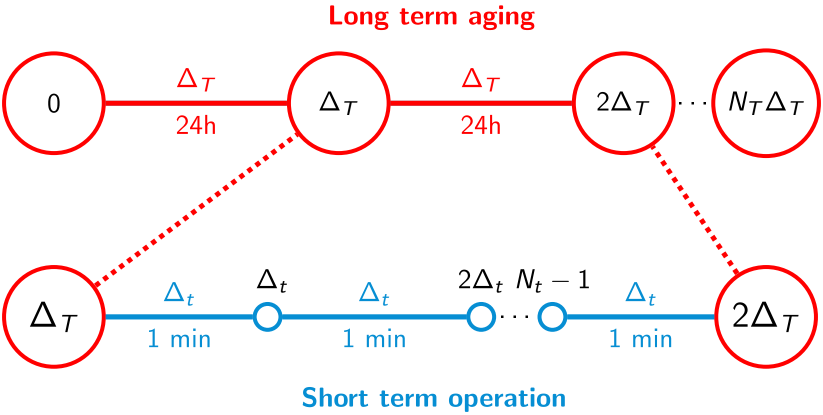
We can optimize battery operations using Stochastic Dynamic Programming

Battery operation impacts long term aging !

Outline

Managing long term batteries aging and renewals

Two time scales

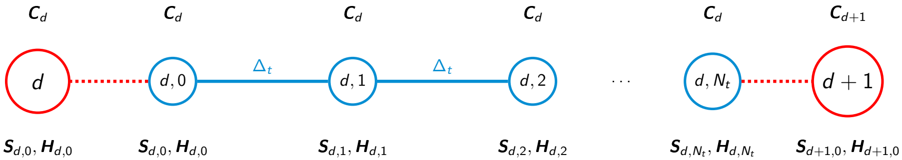


What happens every minute of every day?

Day: d, Minute: m

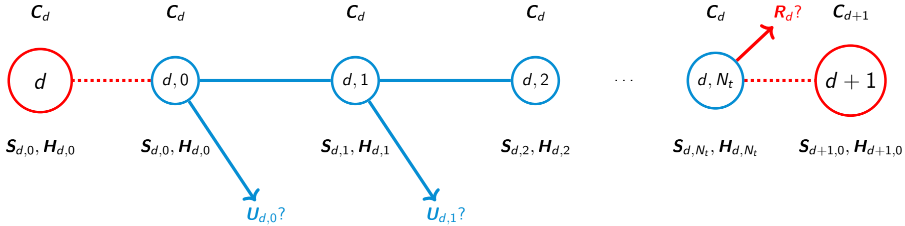
- Battery capacity: $\mathbf{C}_d = \mathbf{C}(d\Delta_T)$
- Battery state of charge: $\mathbf{S}_{d,m} = \mathbf{S}(d\Delta_T + m\Delta_t)$
- Battery state of health: $\mathbf{H}_{d,m} = \mathbf{H}(d\Delta_T + m\Delta_t)$

$\mathbf{S}_{d,m}$ and $\mathbf{H}_{d,m}$ change every minute m

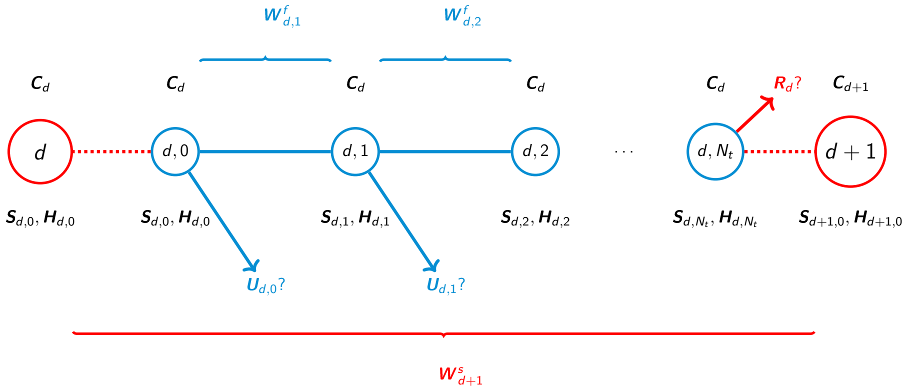


Battery $\mathbf{C}_d$ can be replaced once a day d

We make decisions

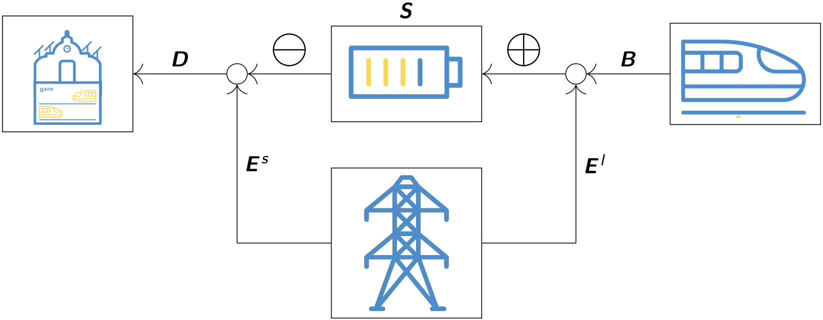


Uncertain events occur



Short term management model

Electrical network representation



Station node

- D: Demand station
- E^s: From grid to station
- $\ominus$: Discharge battery

Subways node

- B: Braking
- E^l: From grid to battery
- $\oplus$: Charge battery

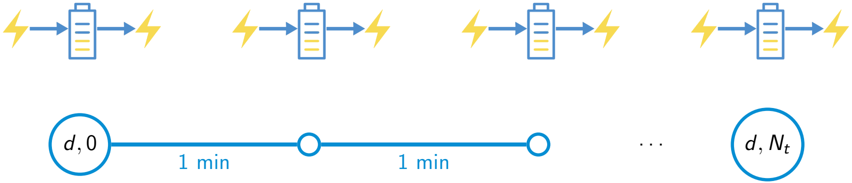
Battery state of charge dynamics

For a given charge/discharge strategy $\boldsymbol{U}$ over a day d:

$$\boldsymbol{S}_{d,m+1} = \boldsymbol{S}_{d,m} - \underbrace{\frac{1}{\rho_d}\boldsymbol{U}^-_{d,m}}_{\ominus} + \underbrace{\rho_c sat(\boldsymbol{S}_{d,m}, \boldsymbol{U}^+_{d,m}, \boldsymbol{B}_{d,m+1})}_{\oplus}$$

with

$$sat(x, u, b) = \min(\frac{S_{max} - x}{\rho_c}, \max(u, b))$$



Battery aging dynamics

For a given charge/discharge strategy $\boldsymbol{U}$ over a day d

$$\boldsymbol{H}_{d,m+1} = \boldsymbol{H}_{d,m} - \frac{1}{\rho_d}\boldsymbol{U}^-_{d,m} - \rho_c sat(\boldsymbol{S}_{d,m}, \boldsymbol{U}^+_{d,m}, \boldsymbol{B}_{d,m+1})$$

Every minute we save energy and money

If we have a battery on day d and minute m we save:

$$p^e_{d,m}\Big(\underbrace{\boldsymbol{E}^s_{d,m+1} + \boldsymbol{E}^l_{d,m+1} - \boldsymbol{D}_{d,m+1}}_{\text{Saved energy}}\Big)$$

$p^e_{d,m}$ is the cost of electricity on day d at minute m

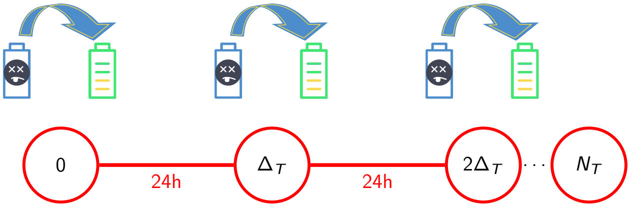
Summary of short term/Fast variables model

We call at day d and minute m:

- fast state variables: $\boldsymbol{X}^f_{d,m} = \begin{pmatrix} \boldsymbol{S}_{d,m} \\ \boldsymbol{H}_{d,m} \end{pmatrix}$
- fast decision variables: $\boldsymbol{U}^f_{d,m} = \begin{pmatrix} \boldsymbol{U}^-_{d,m} \\ \boldsymbol{U}^+_{d,m} \end{pmatrix}$
- fast random variables: $\boldsymbol{W}^f_{d,m} = \begin{pmatrix} \boldsymbol{B}_{d,m} \\ \boldsymbol{D}_{d,m} \end{pmatrix}$
- fast cost function: $L^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$
- fast dynamics: $\boldsymbol{X}^f_{d,m+1} = F^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$
- We define initial value of $\boldsymbol{X}^f_{d,0}$ in the following

Long term management model

We decide our battery purchases at the end of each day



Should we replace our battery $\boldsymbol{C}_d$ by buying a new one $\boldsymbol{R}_d$ or not?

$$\boldsymbol{C}_{d+1} = \begin{cases} \boldsymbol{R}_d, \text{ if } \boldsymbol{R}_d > 0 \\ \boldsymbol{C}_d, \text{ otherwise} \end{cases}$$

paying renewal cost $\boldsymbol{P}_d^b \boldsymbol{R}_d$ at uncertain market prices $\boldsymbol{P}_d^b$

Summary of long term/Slow variables model

We call at day d:

- slow state variables: $\boldsymbol{X}^s_d = (\boldsymbol{C}_d)$
- slow decision variables: $\boldsymbol{U}^s_d = (\boldsymbol{R}_d)$
- slow random variables: $\boldsymbol{W}^s_d = (\boldsymbol{P}^b_d)$
- slow cost function: $L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}) = \boldsymbol{P}^b_d \boldsymbol{R}_d$
- slow dynamics: $\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1})$

Linking the two scales

Slow decisions may impact fast initial state:

$$\boldsymbol{X}^f_{d,0} = \phi_d(\boldsymbol{X}^s_d, \boldsymbol{X}^f_{d-1,N_t})$$

This is not the case here but in general fast variables may impact directly slow dynamics:

$$\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})$$

as well as the slow cost:

$$L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})$$

Defining a two-scale stochastic optimization problem

Two-scale stochastic optimization problem

We minimize fast and slow costs over the long term:

$$\min_{\boldsymbol{X}^f, \boldsymbol{X}^s, \boldsymbol{U}^f, \boldsymbol{U}^s} \mathbb{E}\Big[\sum_{d=0}^{N_T - 1} \Big(\sum_{m=0}^{N_t - 1} L^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1}) \Big) + L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) \Big]$$

$$\boldsymbol{X}^f_{d,m+1} = F^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$$

$$\boldsymbol{X}^f_{d,0} = \phi_d(\boldsymbol{X}^s_d, \boldsymbol{X}^f_{d-1,N_t})$$

$$\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})$$

$$\boldsymbol{U}^f_{d,m} \preceq \mathcal{F}_{d,m}$$

$$\boldsymbol{U}^s_d \preceq \mathcal{F}_{d,N_t}$$

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Information model

$$\mathcal{F}_{d,m} = \sigma \begin{pmatrix} \boldsymbol{W}^{f}_{d',m'},\ d'<d,\ m'\leq N_t+1 \\ \boldsymbol{W}^{s}_{d'},\ d'\leq d \\ \boldsymbol{W}^{f}_{d,m'},\ m'\leq m \end{pmatrix} = \sigma \begin{pmatrix} \text{previous days fast noises} \\ \text{previous days slow noises} \\ \text{current day previous minutes fast noises} \end{pmatrix}$$

Fast information accumulation:

$$\mathcal{F}_{d,m} = \mathcal{F}_{d,0} \vee \sigma(\boldsymbol{W}^{f}_{d,1:m})$$

Slow information implies a jump between d, N_t and $d+1, 0$

$$\mathcal{F}_{d+1,0} = \mathcal{F}_{d,N_t} \vee \sigma(\boldsymbol{W}^{s}_{d+1})$$

Two-scale stochastic optimal control reformulation

With

$$\boldsymbol{X}_d = (\boldsymbol{X}^f_{d-1,N_t}, \boldsymbol{X}^s_d)$$
$$\boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d)$$
$$\boldsymbol{W}_d = (\boldsymbol{W}^f_{d-1,:}, \boldsymbol{W}^s_d)$$

We show that we can reformulate the problem as:

$$\min_{\boldsymbol{X},\boldsymbol{U}} \mathbb{E}\Big[\sum_{d=0}^{N_T-1} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})\Big]$$
$$\boldsymbol{X}_{d+1} = F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})$$
$$\boldsymbol{U}^f_{d,m} \preceq \mathcal{F}_{d,m}$$
$$\boldsymbol{U}^s_d \preceq \mathcal{F}_{d,N_t}$$

where the non-anticipativity constraints are not standard

Outline

How to decompose a two-scale stochastic optimization problem into:

a short term optimization problem

and

a long term optimization problem?

Slow value function and Bellman equation

Slow value function

Every "day" d_0 we can define a slow value function

$$V_{d_0}(x_{d_0}) = \min_{\boldsymbol{X},\boldsymbol{U}} \mathbb{E}\left[\sum_{d=d_0}^{N_T-1} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})\right]$$

$$\boldsymbol{X}_{d+1} = F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})$$

$$\boldsymbol{U}^f_{d,m} \preceq \mathcal{F}_{d,m}$$

$$\boldsymbol{U}^s_d \preceq \mathcal{F}_{d,N_t}$$

$$\boldsymbol{X}_{d_0} = x_{d_0}$$

It satisfies a Bellman equation

Assuming independence of the noises $\boldsymbol{W}_d$

$$V_d(x_d) = \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + V_{d+1}\Big(F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})\Big)\right]$$

$$\text{s.t } \boldsymbol{U}_d = (\boldsymbol{U}^s_d, \boldsymbol{U}^f_{d,:})$$

$$\boldsymbol{U}^f_{d,m} \preceq \boldsymbol{W}^f_{d,1:m}$$

$$\boldsymbol{U}^s_d \preceq \boldsymbol{W}^f_{d,1:N_t}$$

with non standard non-anticipativity constraints as well

Slow features may depend on fast final state only

Sometimes the slow scale cost simplifies:

$$L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) = L^s_d(x^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,N_t})$$

As the aggregated dynamics:

$$F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) = F_d(x^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,N_t})$$

Leading to:

$$\begin{aligned} V_d(x_d) = \min_{\boldsymbol{U}^s_d, \boldsymbol{U}^f_{d,:}, \boldsymbol{X}^f_{d,N_t}} \quad & \mathbb{E}\Big[L^f_d(\boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) + L^s_d(x^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,N_t}) \\ & + V_{d+1}\Big(F_d(x^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,N_t})\Big)\Big] \\ \text{s.t } & \boldsymbol{X}^f_{d,N_t} = F^f_{d,:}(\boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) \\ & \boldsymbol{X}^f_{d,0} = \phi_d(x_d) \\ & \boldsymbol{U}^f_{d,m} \preceq \boldsymbol{W}^f_{d,1:m} \\ & \boldsymbol{X}^f_{d,N_t} \preceq \boldsymbol{W}^f_{d,1:N_t} \end{aligned}$$

Bellman equation decomposition

Bilevel Stochastic Dynamic Programming (BSDP)

Using expectation linearity and primal decomposition we show that:

$$
\begin{aligned}
V_d(x_d) = \min_{\boldsymbol{U}^s_d, \hat{\boldsymbol{X}}} \; & V^f_{d,0}(\phi_d(x_d), \hat{\boldsymbol{X}}) + \mathbb{E}\Big[L^s_d(x^s_d, u^s_d, \boldsymbol{W}^s_{d+1}, \hat{\boldsymbol{X}}) \\
& \qquad + V_{d+1}\Big(F_d(x^s_d, u^s_d, \boldsymbol{W}^s_{d+1}, \hat{\boldsymbol{X}}) \Big) \Big] \\
\text{s.t } & \hat{\boldsymbol{X}} \preceq \boldsymbol{W}^f_{d,:} \\
& \boldsymbol{U}^s_d \preceq \boldsymbol{W}^f_{d,:}
\end{aligned}
$$

We notice that this problem involves only slow variables, fast variables being hidden in the fast value function $V^f_{d,0}$.

The problem is decomposed into a high level slow problem and a low level fast problem.

Fast problem parameterized value function

On day d we define the fast initial value function:

$$V^f_{d,0} : \mathbb{X}^f_{d,0} \times L^0(\Omega, \mathcal{F}, \mathbb{P}) \to \mathbb{R}$$

such as $\forall x^f_{d,0} \in \mathbb{X}^f_{d,0},\ \forall \hat{\boldsymbol{X}} \preceq \boldsymbol{W}^f_{d,:}$

$$\begin{aligned} V^f_{d,0}(x^f_{d,0}, \hat{\boldsymbol{X}}) = \min_{\boldsymbol{U}^f_{d,:}} \ & \mathbb{E}\left[L^f_d(x^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})\right] \\ \text{s.t } & F^f_{d,:}(x^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) = \hat{\boldsymbol{X}} \\ & \boldsymbol{U}^f_{d,m} \preceq \boldsymbol{W}^f_{d,1:m} \end{aligned}$$

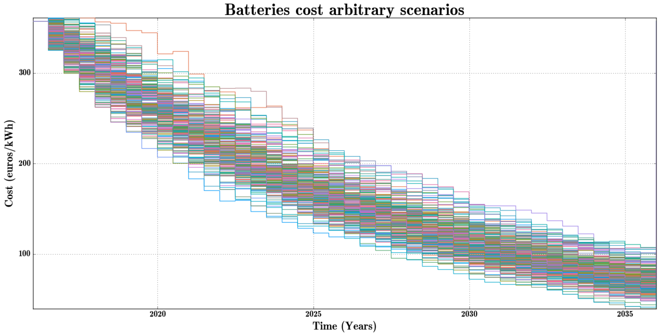
Practical tricks

- $\hat{\boldsymbol{X}}$ measurability restriction: $\hat{\boldsymbol{X}} \preceq \{\emptyset, \Omega\}$
- Final state constraint of short term problem relaxation: $F^f_{d,:}(\boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:}) \geq \hat{\boldsymbol{X}}$.
  We preserve equality when we have monotonicity of small scale value function in the fast state variables (Heymann et al. [2])
- Compute a limited number a fast value functions exploiting fast variables seasonalities $V^f_{d,0}(x^f_{d,0}, \hat{\boldsymbol{X}}) = V^f_{0,0}(x^f_{d,0}, \hat{\boldsymbol{X}})$

Preliminary numerical results

Synthetic data

- Maximum exangeable energy: model proposed in Haessig et al. [1]
- Discount rate: 4.5%
- Batteries cost stochastic model: synthetic scenarios that approximately coïncides with market forecasts



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Comparison of 3 strategies

We compare 3 investement strategies over 20 years, 100 $\boldsymbol{C}^b$ scenarios, 1 single capacity (80 kWh)

Straightforward approach, investment/control independence:

- Strategy NPV : Buy now, replace battery when dead, no aging control

Bilevel Stochastic Dynamic Programming:

- Strategy NPVA: Buy now, replace battery when dead, control aging
- Strategy FNPVA: Buy anytime, replace battery anytime, control aging

Objective: maximize revenues over 20 years

Preliminary results

- NPV = -7000 euros ⇒ do not invest!
- NPVA = +12000 euros ⇒ do not strain your first batteries!
- FNPVA = +33000 euros ⇒ start investment in 2020 and do not strain your first batteries!

	SDP	**BSDP**
Offline comp. time	∞ (out of memory)	16min
Online comp. time	?	[0s,1s]
Simulation comp. time	?	[20s,30s]
Lower bound	?	+38k

In Julia with a Core I7, 2.6 Ghz, 8Go ram + 12Go swap SSD

Conclusion

Our study leads to the following conclusions:

- Controlling aging is highly important
- BSDP provides encouraging results
- BSDP can be used for aging aware intraday control
- Classical Net Present Value and Free Net Present Value lead to different conclusions
- Free Net Present Value uses a more accurate model of the investment management

Ongoing work

We are now focusing on:

- Confirming, developing and improving BSDP results
- Improving risk modelling
- Improving batteries cost stochastic model
- Improving aging model
- Include environmental incentives (particulate matters)
- Apply the method to more complex energy efficiency investments

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