Nonlinear Analysis: Geometric, Variational and Dispersive aspects

in a celebration of Michał Kowalczyk's 60th birthday
Center for Mathematical Modeling (CMM), Universidad de Chile
Santiago – Chile

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1 Estimates on the L^2 norm of the positive solutions of a two parameter family of nonlinear PDE´s of the TFW type.

Speaker: Rafael Benguria
Affiliation(s): Pontificia Universidad Católica de Chile

Abstract: In this talk I consider the two parameter family of PDE´s (generalized TFW equation):

$$-\Delta u + (\gamma u^{2p-2} - \phi)u = 0,$$

on $\mathbb{R}^3$, where,

$$\phi(x) = \frac{Z}{|x|} - \int_{\mathbb{R}^3} \frac{u^2(y)}{|x-y|}\, dy.$$

Here, $Z > 0$ is fixed and $\gamma \geq 0$, and $1 < p$. The case $p = 5/3$ corresponds to the Thomas-Fermi-von Weizs"acker equation in the atomic case, which was first studied by R. Benguria, H. Brezis, and E.H. Lieb in 1981. The case $\gamma = 0$ corresponds to the Hartree equation. Here we are interested in estimating the so called "excess charge´´, given by $Q = \int_{\mathbb{R}^3} \|u\|_2^2 - Z$, for different values of p and γ, where u is the positive solution of the generalized TFW equation. This is joint work with Heinz Siedentop (Ludwig Maximilians University).

2 Topological transitions in an oscillatory driven liquid crystal cell

Speaker: Marcel Clerc
Affiliation(s): Universidad de Chile

Abstract: In this talk, we present topological states of matter in a system with injection and dissipation of energy using oscillatory forcing. In an experiment involving a liquid crystal cell under a low-frequency oscillatory electric field, we observe a transition from a non-vortex state to a state where vortices persist, a topological transition. Depending on the period and the type of force, the vortices self-organize, forming square lattices, glassy states, and disordered vortex structures. The bifurcation diagram is characterized experimentally. A continuous topological transition is observed for the sawtooth and square forcings. The scenario changes dramatically for sinusoidal forcing where the topological transition is discontinuous, accompanied by serial transitions between square and glassy vortex lattices. Based on a stochastic amplitude equation, we recognize the origin of the transition as the balance between stochastic creation and deterministic annihilation of vortices. Numerical simulations show topological transitions and the emergence of a square vortex lattice. Our results show that the matter maintained out of equilibrium by means of the temporal modulation of parameters can exhibit exotic states.

3 From PDE to school mathematics

Speaker: Patricio Felmer
Affiliation(s): Universidad de Chile

Abstract: In this conference, I will present my experience in transit from PDEs as a research area in mathematics to school mathematics as a research and development area, the current center of my academic work. I will explain, what we are doing and the main challenges we face with the ARPA Initiative team in teacher professional development.

4 Curvature and geometric obstructions for phase transitions

Speaker: Pedro Gaspar
Affiliation(s): Pontificia Universidad Católica de Chile
Abstract: The Allen-Cahn equation is a semilinear partial differential equation that serves as a mathematical model for the evolution of phase separation and pattern formation phenomena, and whose solutions display striking geometric features. In this talk, we present new examples concerning the (non)existence of certain equilibrium solutions of this PDE in connection to degenerate minimal hypersurfaces. We also discuss how these examples relate to a theorem by Frankel about minimal surfaces in spaces of nonnegative Ricci curvature and to an associated rigidity property.

5 Beyond the Gidas-Ni-Nirenberg theorem: the case of nonconvex domains

Speaker: Massimo Grossi
Affiliation(s): Sapienza University of Rome

Abstract: The classical Gidas-Ni-Nirenberg theorem says that the positive solutions of a large class of elliptic equations in convex and symmetric domain admits exactly one critical point. Outside of the convex setting it is well known that it is not true anymore. In this talk we explore the problem to compute the number of critical points of a solution in any planar domain in the particular case of the Gelfand Problem.

6 The Allen-Cahn equation on surfaces

Speaker: Frank Pacard
Affiliation(s): École Polytechnique

Abstract: It is well known that there are are strong links between the solutions of the Allen-Cahn equation and minimal submanifolds. In the special case where the underlying manifold is a compact surface, I will present the constriction of solutions for the Allen-Cahn equation whose nodal set converges to the union of immersed geodesics as the parameter in the Allen-Cahn equation tends to 0. This is a joint work with Yong Liu and Juncheng Wei.

7 Free boundary problems: Liouville equation and Bose-Einstein condensates

Speaker: Angela Pistoia
Affiliation(s): Sapienza University of Rome

Abstract: The first result claims the existence of solutions with infinite mass for the Liouville equation with Dirichlet boundary conditions in a two dimensional doubly connected domain. The key ingredient in the construction is the solution of a suitable free boundary problem. The method of the proof inspired the second result which states the existence of a solution of a two component system of coupled non linear Schrödinger equations modeling the phase separation in the binary mixture of Bose–Einstein condensates. The results have been obtained in collaboration with Michał Kowalczyk and Giusi Vaira.

8 Stability of Sobolev Inequalities and Sharp Quantitative Estimates of Struwe's Decomposition

Speaker: Juncheng Wei
Affiliation(s): University of British Columbia

Abstract: In this talk I will discuss the stability of Sobolev inequality

$$\|\nabla u\|_{L^2}^2 - S_n \|u\|_{L^{2n/(n-2)}}^2 \geq 0$$

from the critical point theory. Suppose $u \in D^{1,2}$, in a seminal work, Struwe proved that if $u \geq 0$ and $\Gamma(u) = \|\Delta u + u^{\frac{n+2}{n-2}}\|_{H^{-1}} \to 0$ then $\text{dist}(u, T) \to 0$,where $dist(u, T)$ denotes the $D^{1,2}$-distance of u from the manifold of sums of Talenti bubbles. Ciraolo,Figalli and Maggi obtained the first quantitative version of Struwe's decomposition with one bubble in all dimensions, namely $dist(u, T) \leq C\Gamma(u)$. For Struwe's decomposition with two or more bubbles, Figalli and Glaudo showed a striking dimensional dependent quantitative estimate, namely $dist(u, T) \leq \Gamma(u)$ when $3 \leq n \leq 5$ while this is false for $n \geq 6$. In this talk, I will present an optimal estimate $dist(u, T) \leq C\Gamma(u)|\log \Gamma(u)|^{1/2}$ if $n = 6$, and $dist(u, T) \leq C|\Gamma(u)|^{\frac{n+2}{2(n-2)}}$ if $n \geq 7$. Furthermore, we show that this inequality is sharp. Extensions to Caffarelli-Kohn-Nirenberg and Harmonic Maps will also be discussed. (Joint work with Deng and Sun.)

9 Stability analysis of entire radial solutions to an anisotropic Ginzburg-Landau equation

Speaker: Andrés Zuñiga
Affiliation(s): Universidad de O'Higgins (UOH)

Abstract: We study the linear stability of entire radial solutions $u(re^{i\theta}) = f(r)e^{i\theta}$ with positive increasing profile $f(r)$ to the anisotropic Ginzburg-Landau equation $-\Delta u - \delta(\partial_x + i\partial_y)^2\bar{u} = (1 - |u|^2)u$, $-1 < \delta < 1$, which arises in various liquid crystals models. In the isotropic case $\delta = 0$, Mironescu showed that such solution is nondegenerately stable. We prove stability of this radial solution in the range $\delta \in (\delta_1, 0]$ for some $-1 < \delta_1 < 0$ and instability outside this range. In strong contrast with the isotropic case, stability with respect to higher Fourier modes is not a direct consequence of stability with respect to lower Fourier modes. In particular, in the case where $\delta \approx -1$, lower modes are stable and yet higher modes are unstable. Joint work with Xavier Lamy (Université Paul Sabatier - Toulouse III). A.Z. was supported by ANID Chile under the grant FONDECYT Iniciación 11201259.

10 Stability results for (log) Sobolev inequalities

Speaker: Jean Dolbeault
Affiliation(s): Université Paris Dauphine

Abstract: This lecture is devoted to recent stability results with explicit, dimensionally sharp constants and optimal norms for the Sobolev inequality and for the Gaussian logarithmic Sobolev inequality obtained

in collaboration with M.J. Esteban, A. Figalli, R.L. Frank, and M. Loss. The stability for the Gaussian logarithmic Sobolev inequality can be obtained either as a byproduct of the stability for the Sobolev inequality or by a direct proof.