

# Macroecological laws and emerging biological patterns

A conceptual approach

Emanuele Pigani - PhD student SZN  
Naples and LIPh Lab Padova (Italy)

## Towards a modern analysis of omics data of the Ocean - workshop

Valparaiso (Chile)  
May 18<sup>th</sup>, 2023



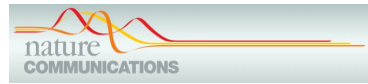
# Macroecological laws on the seascape

ARTICLE

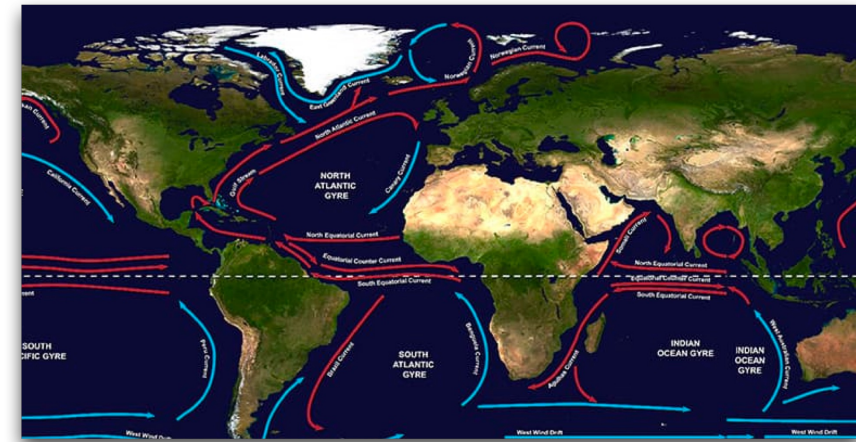
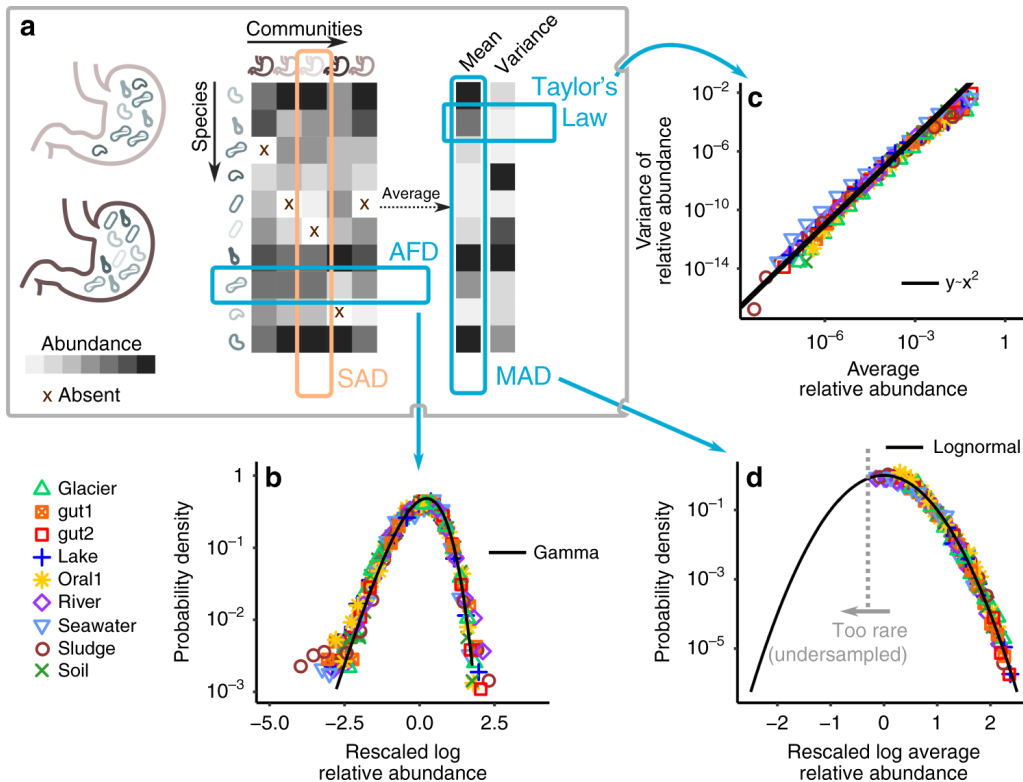
<https://doi.org/10.1038/s41467-020-18529-y> OPEN

## Macroecological laws describe variation and diversity in microbial communities

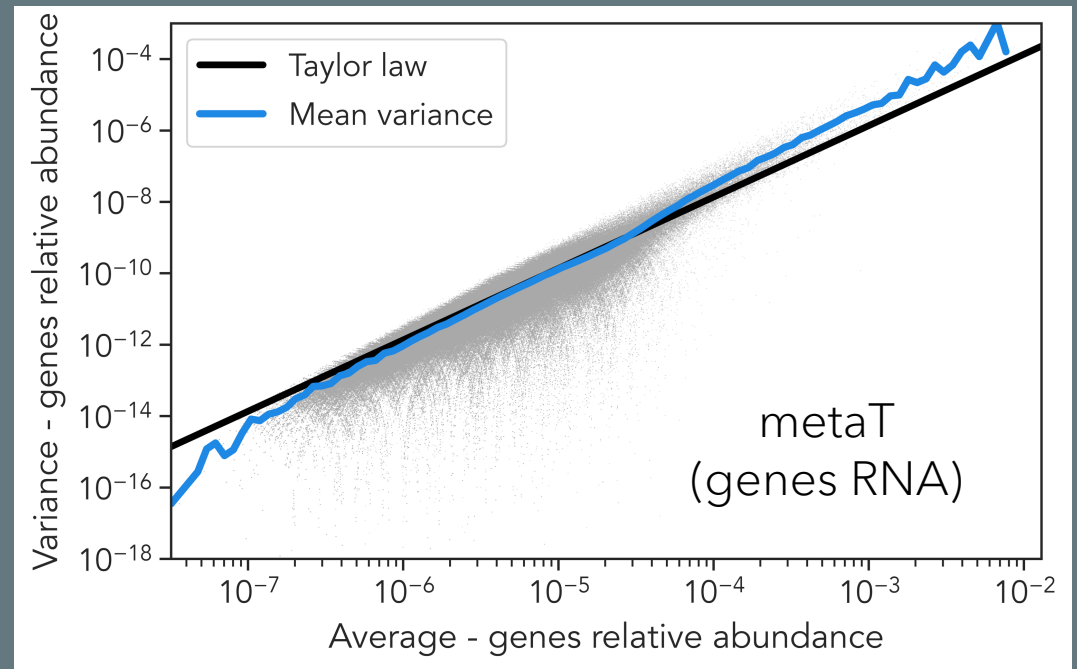
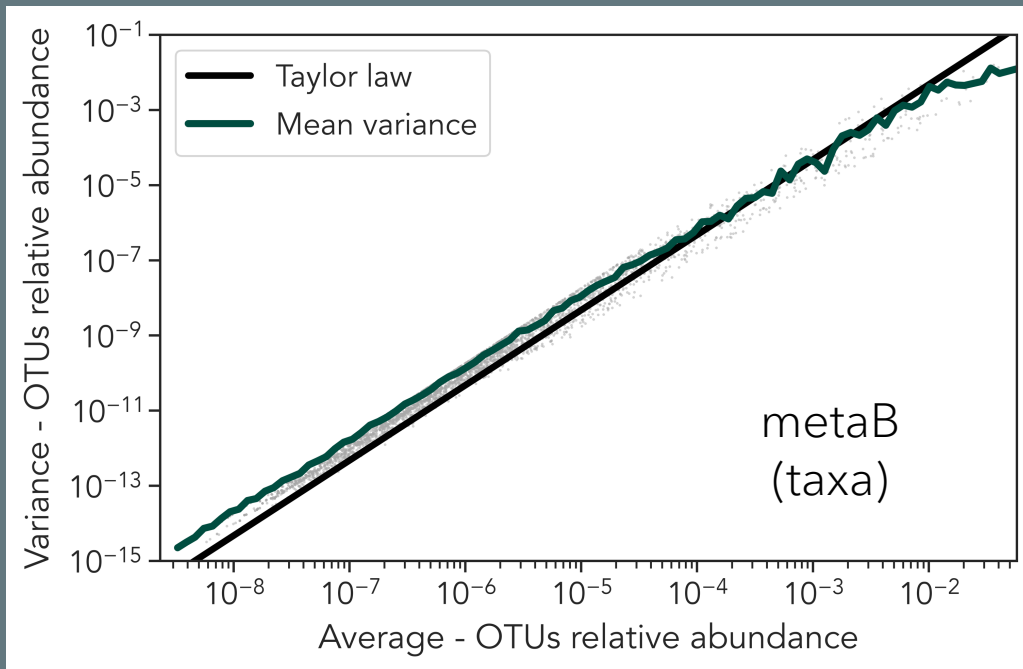
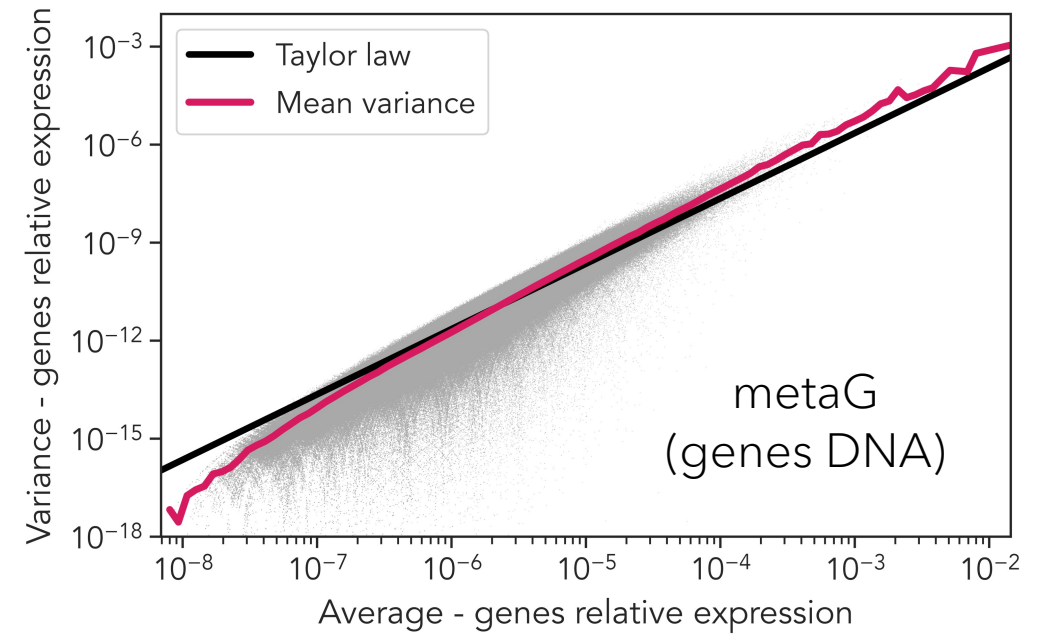
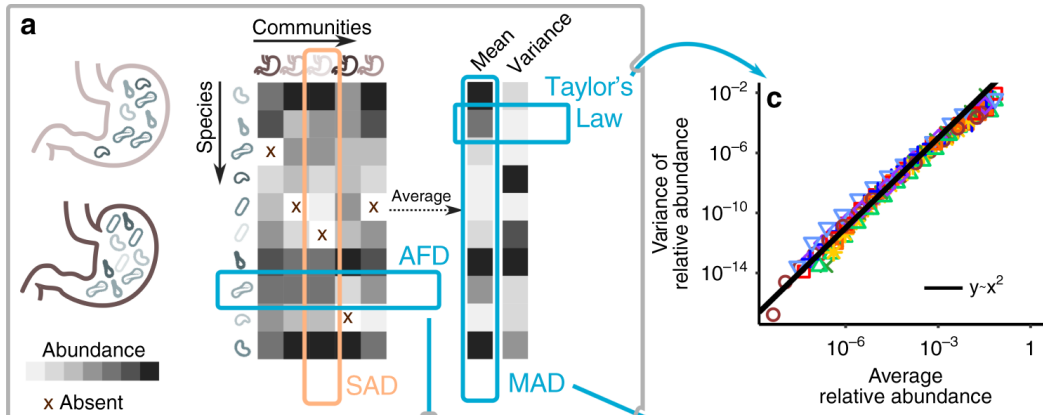
Jacopo Grilli <sup>1,2</sup>

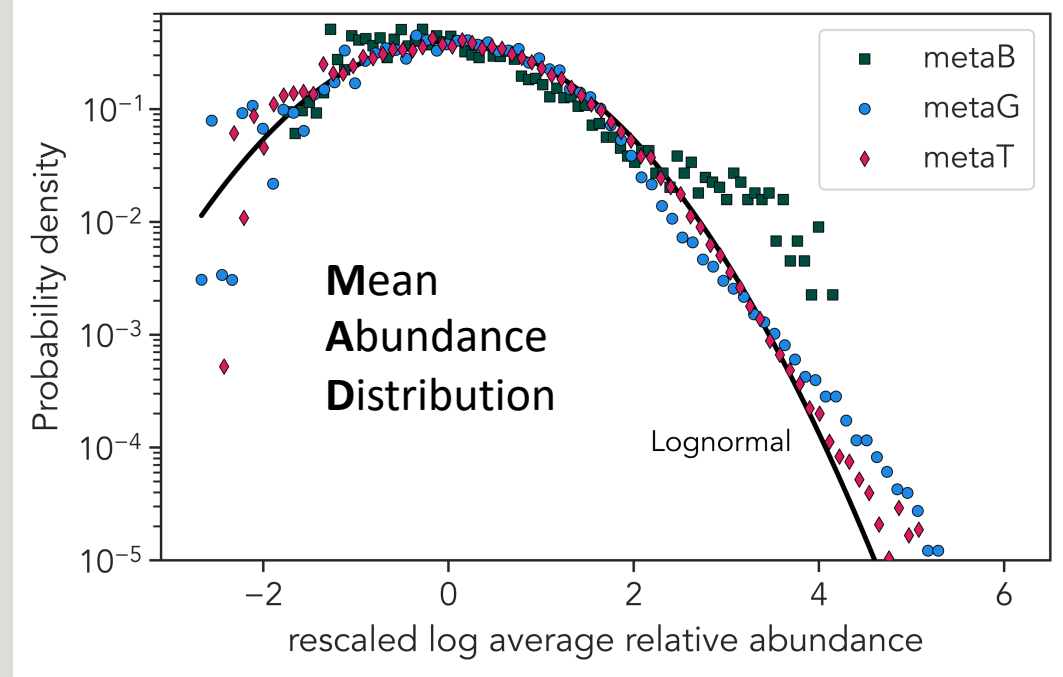
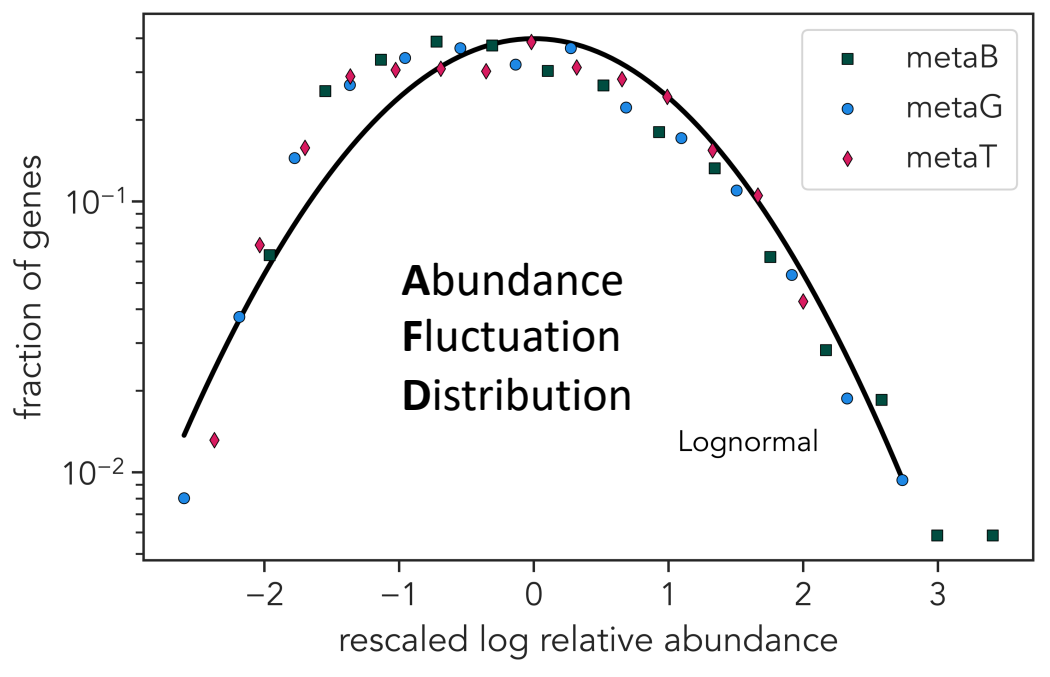


What is the taxonomical and functional impact of the seascape to the communities at macroecological level?

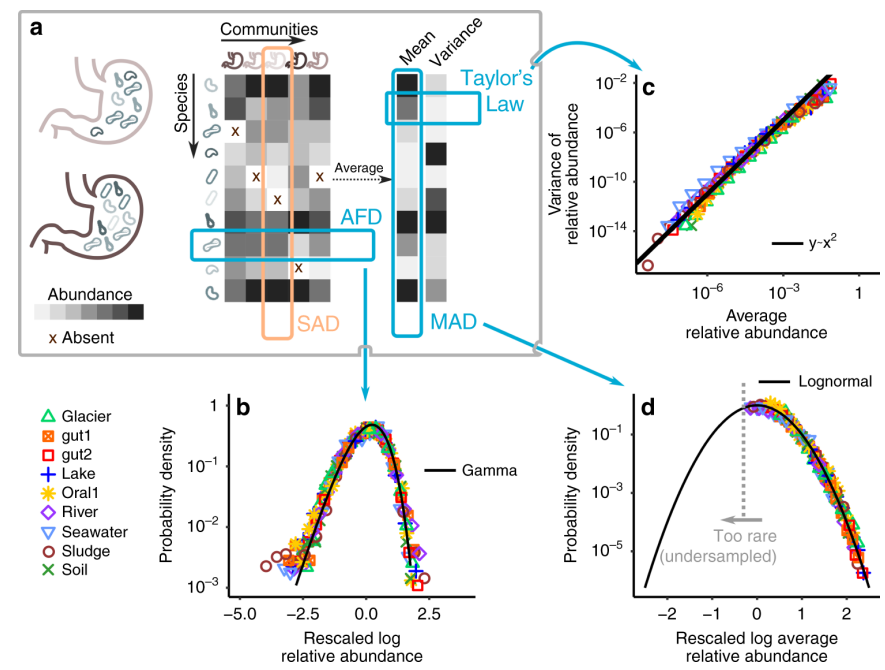


# Taylor's law for diatoms meta-omics

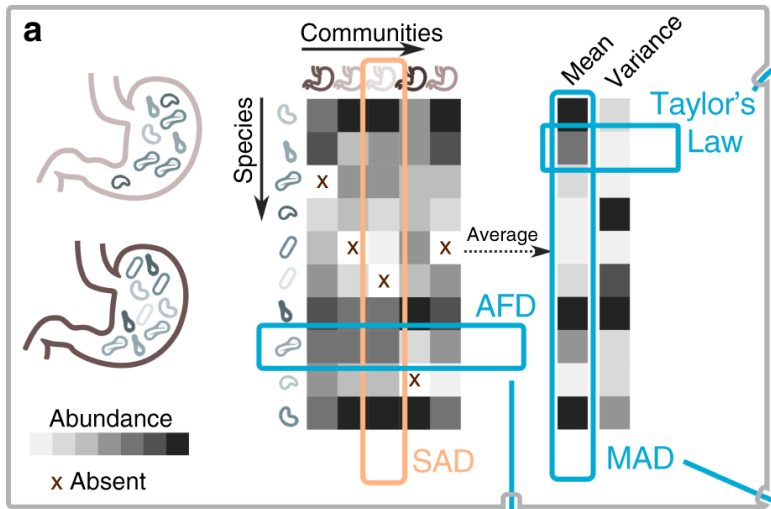




# AFD & MAD



# SAD & RAD



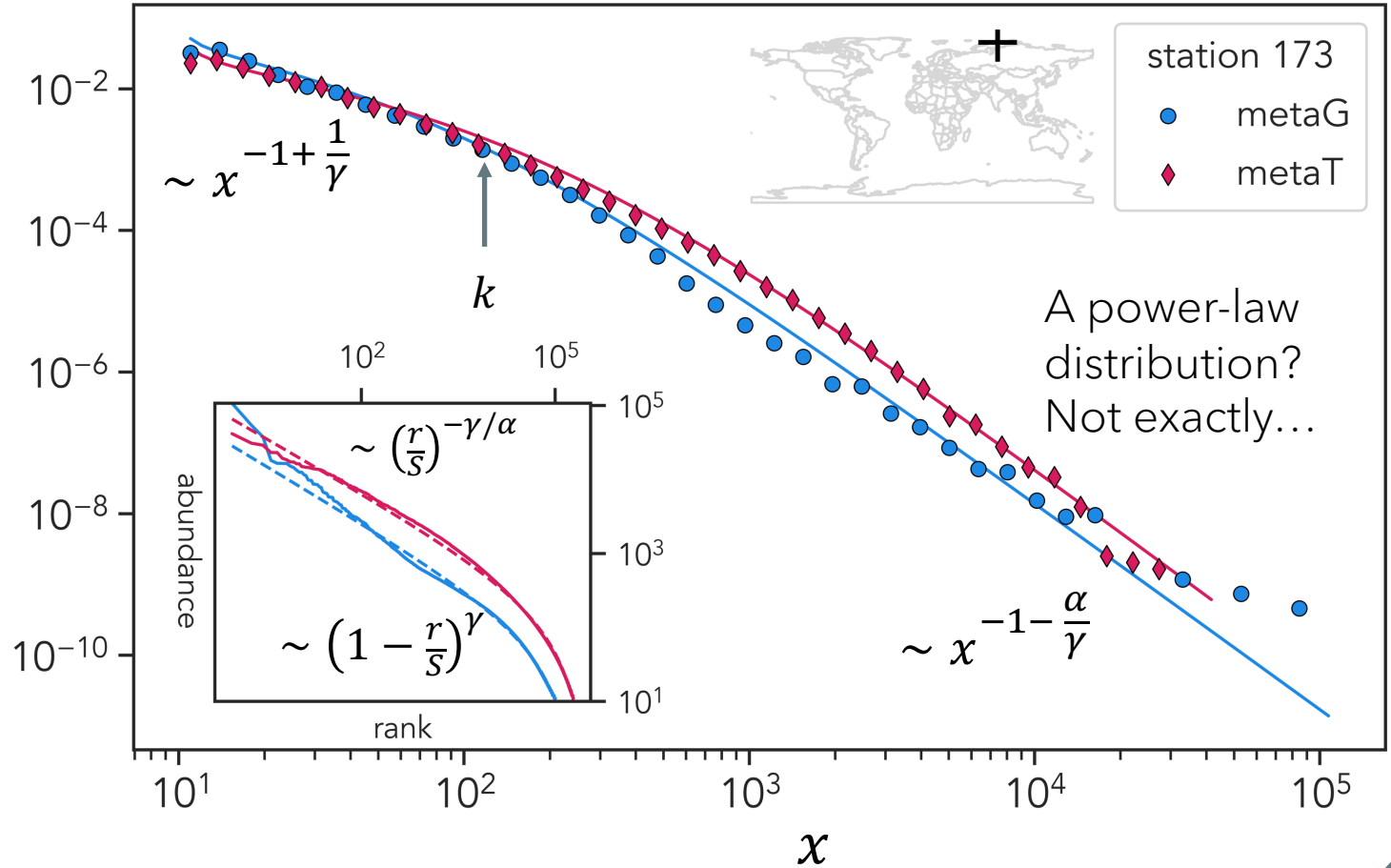
Properties:

- Two power-law regimes separated by a scale  $k$ ;
- Simple power-law distribution (Pareto) in the limit  $\gamma = 1$  and  $\mu = 0$ .

Pareto IV distribution:

$$P(x|k, \alpha, \gamma, \mu) = \frac{\alpha}{\gamma k} \left[ 1 + \left( \frac{x-\mu}{k} \right)^{1/\gamma} \right]^{-1-\alpha} \left( \frac{x-\mu}{k} \right)^{-1+1/\gamma}$$

$P(x)$



- Analytical formula for the Rank Abundance distribution:  $RAD(r) = \mu + k \left[ \left(\frac{r}{s}\right)^{-1/\alpha} - 1 \right]^\gamma$

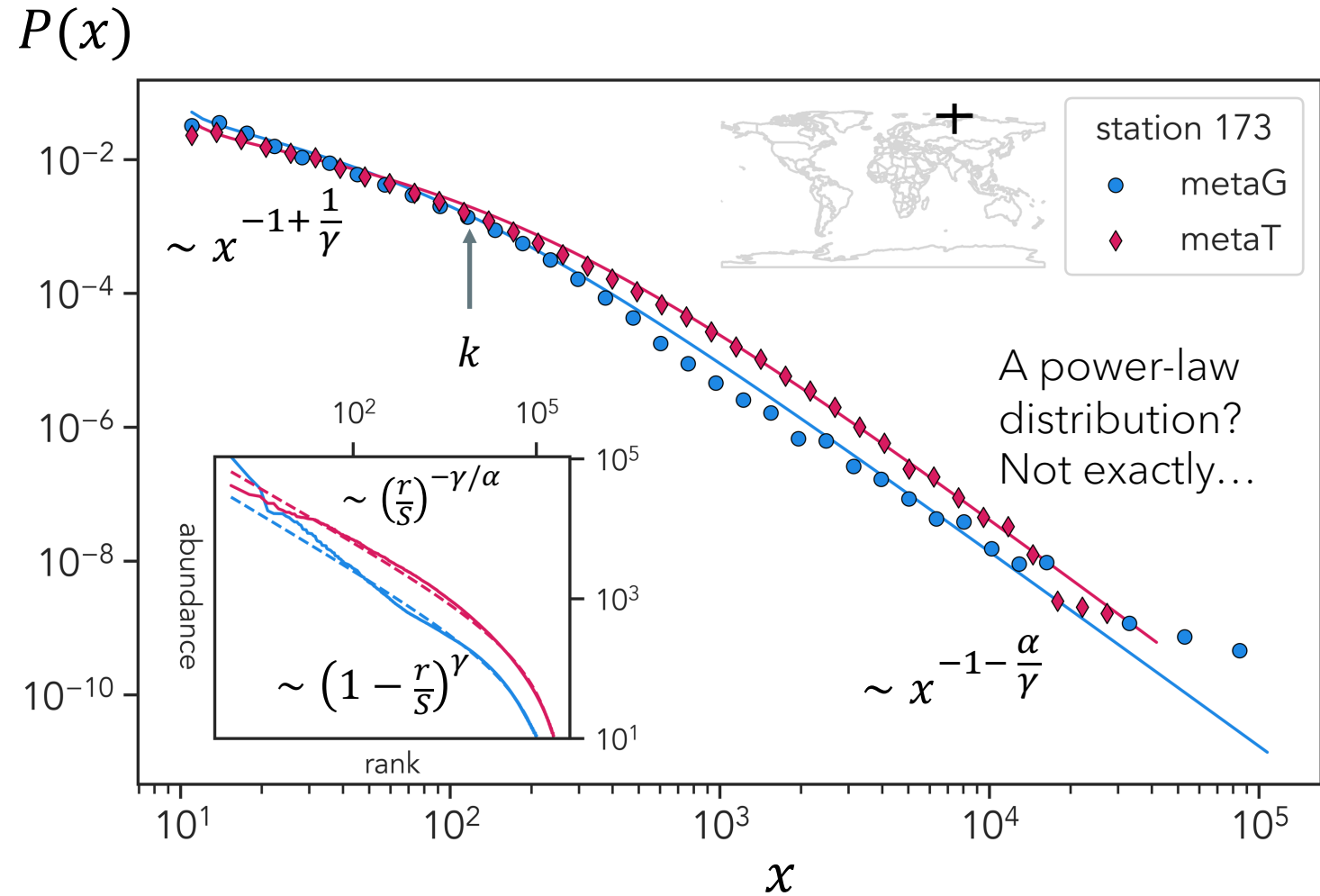
# SAD & RAD

Open questions:

- Is there a **relation** between **metaG** and **metaT**?
- Why the Pareto IV? Is there a **generative model**?
- What is the **biological meaning** of the **parameters**?
- Does it work also for **transcriptomics data**?
- Do we have to take it into account in the **data normalization**?

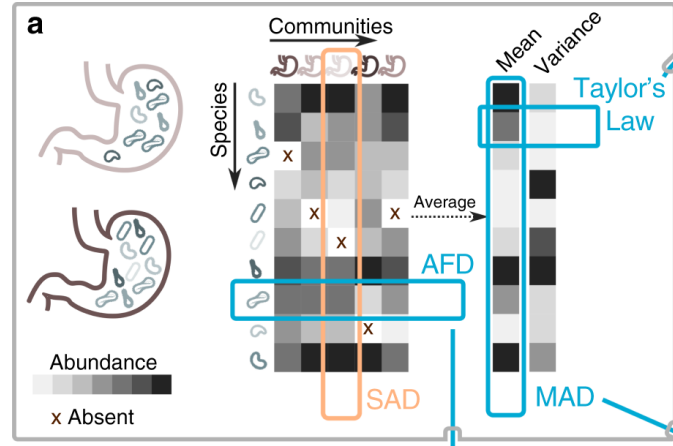
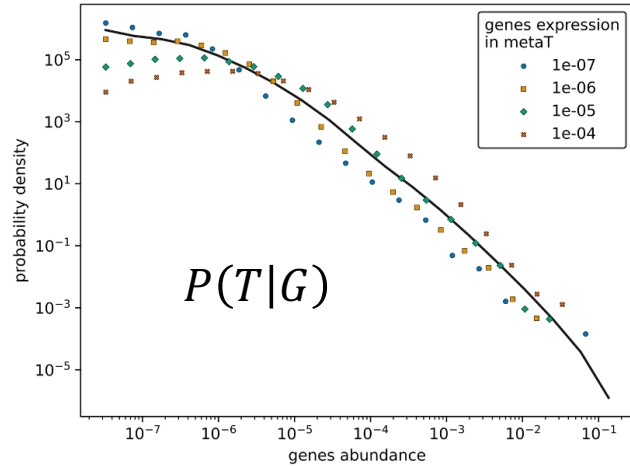
Pareto IV distribution:

$$P(x|k, \alpha, \gamma, \mu) = \frac{\alpha}{\gamma k} \left[ 1 + \left( \frac{x-\mu}{k} \right)^{1/\gamma} \right]^{-1-\alpha} \left( \frac{x-\mu}{k} \right)^{-1+1/\gamma}$$

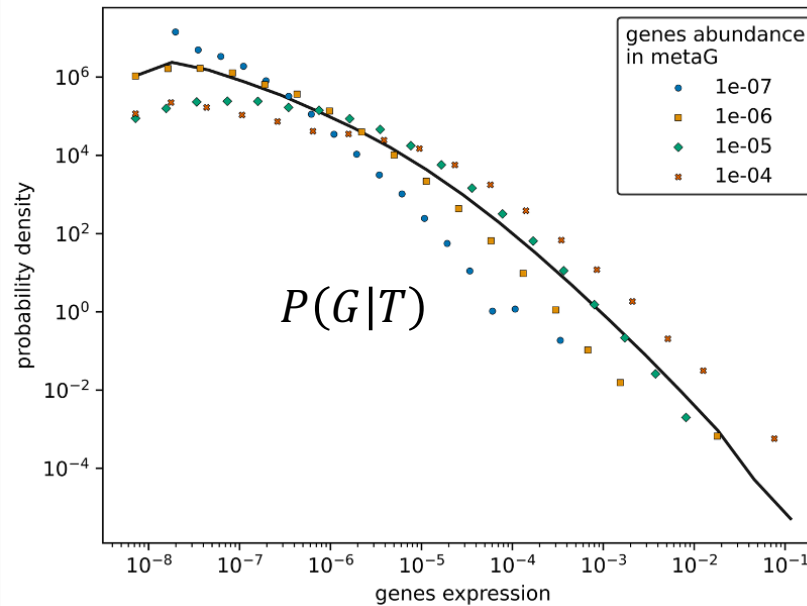
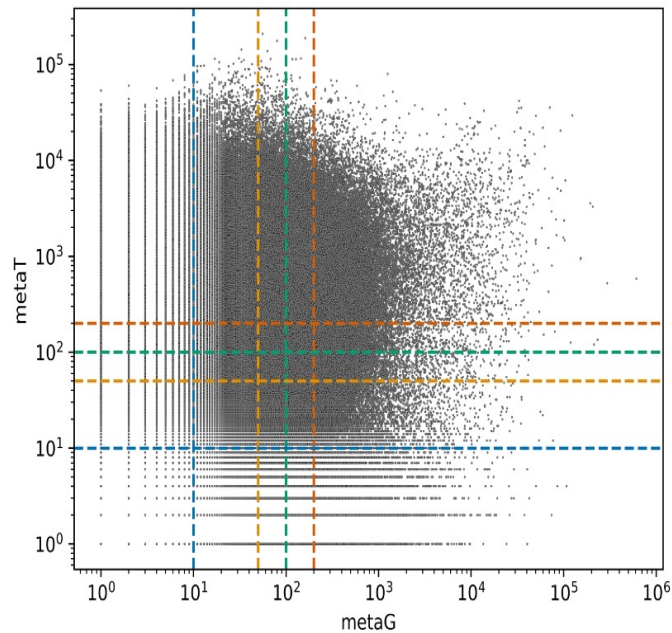


- Analytical formula for the Rank Abundance distribution:  $RAD(r) = \mu + k \left[ \left(\frac{r}{s}\right)^{-1/\alpha} - 1 \right]^\gamma$

# A relation between metaG and metaT?

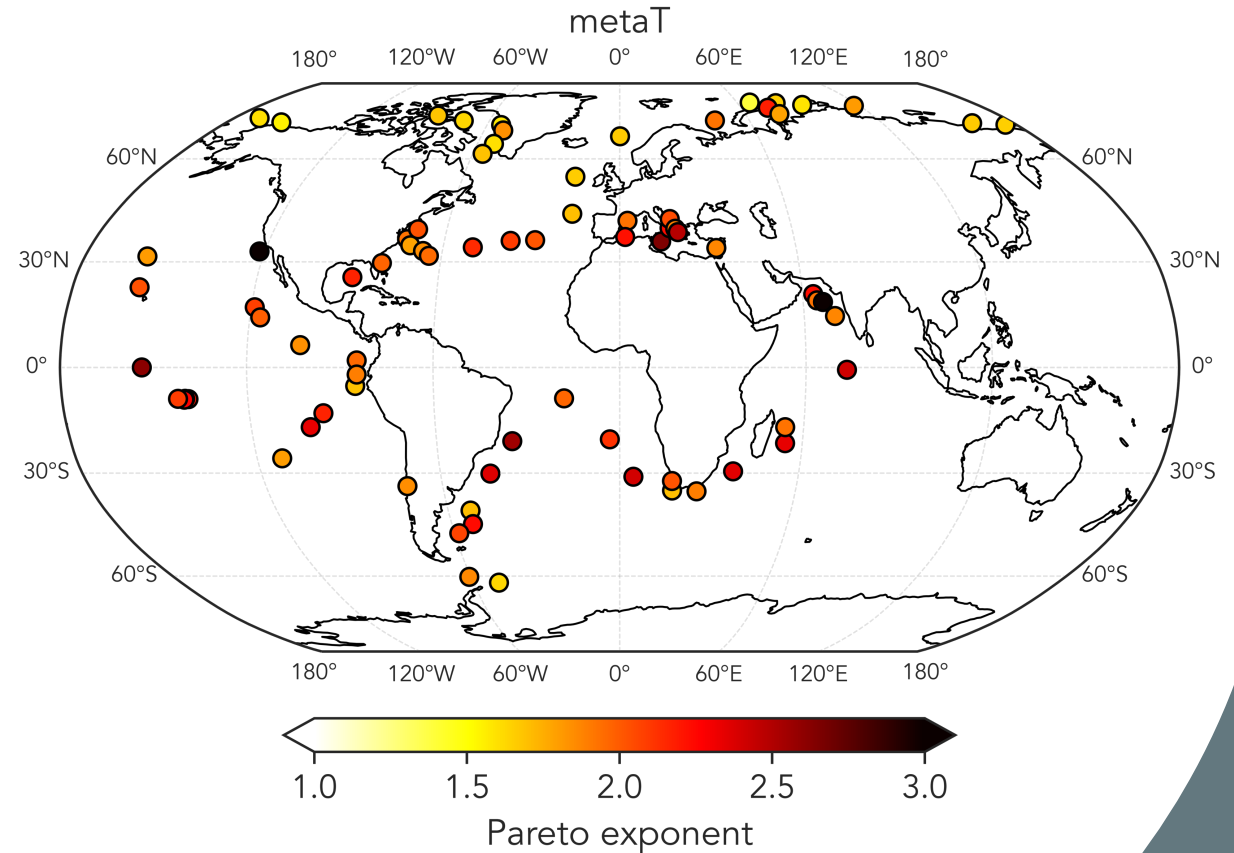
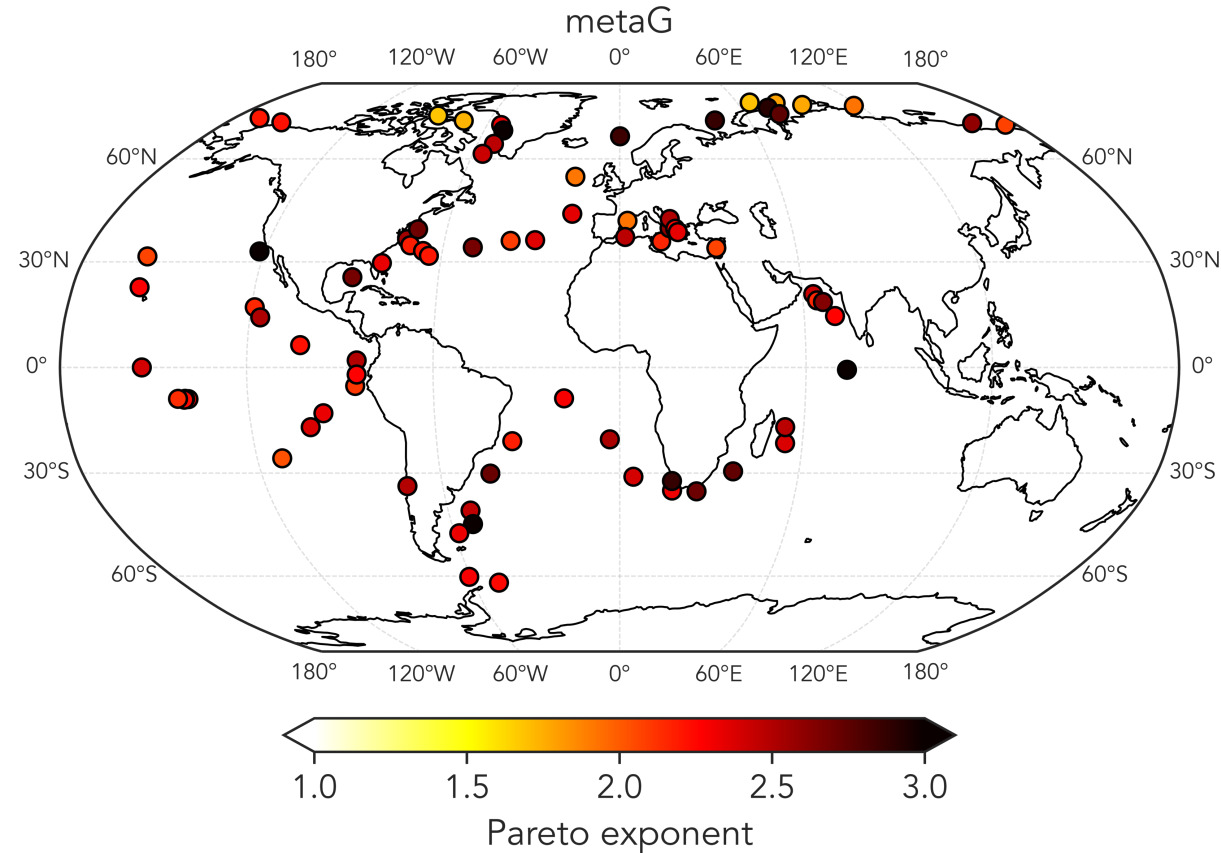


Is a convolution possible?  
It seems difficult...



# A relation between metaG and metaT?

$$P(x \gg k) \sim x^{-1-\frac{\alpha}{\gamma}}$$



metaT exponents display more **heterogeneity** and **regionality**

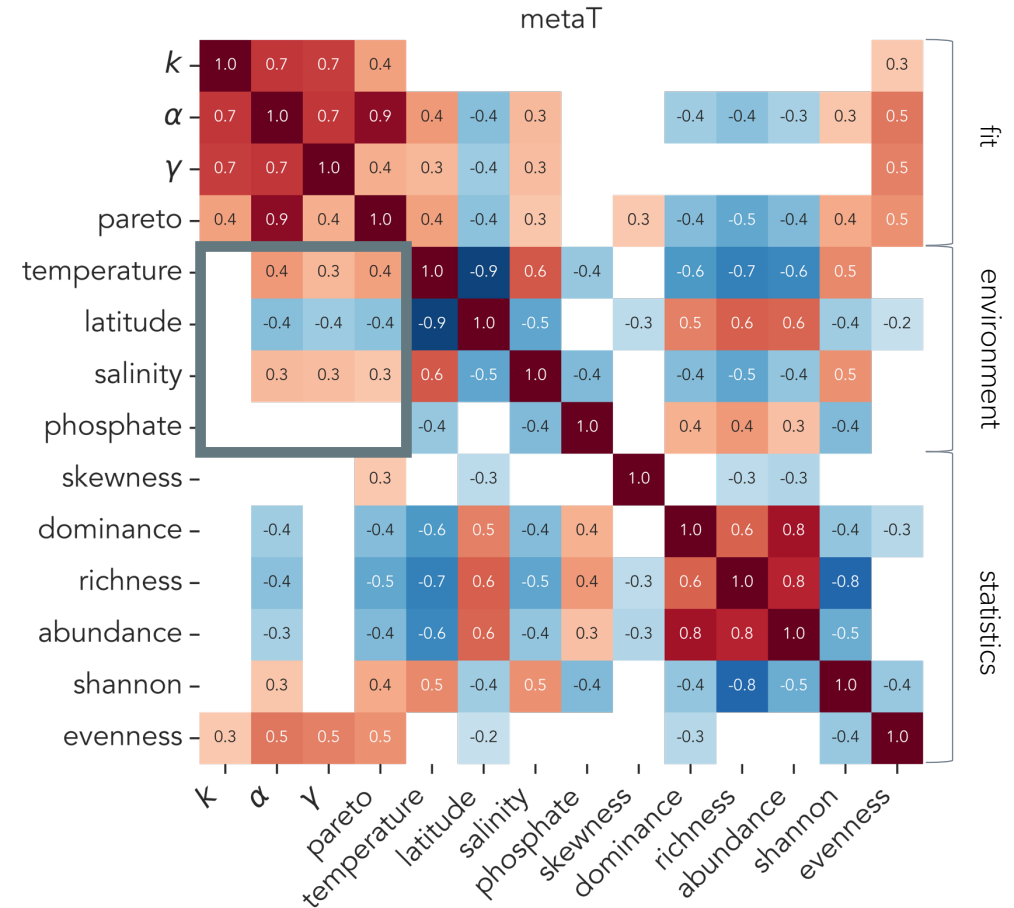
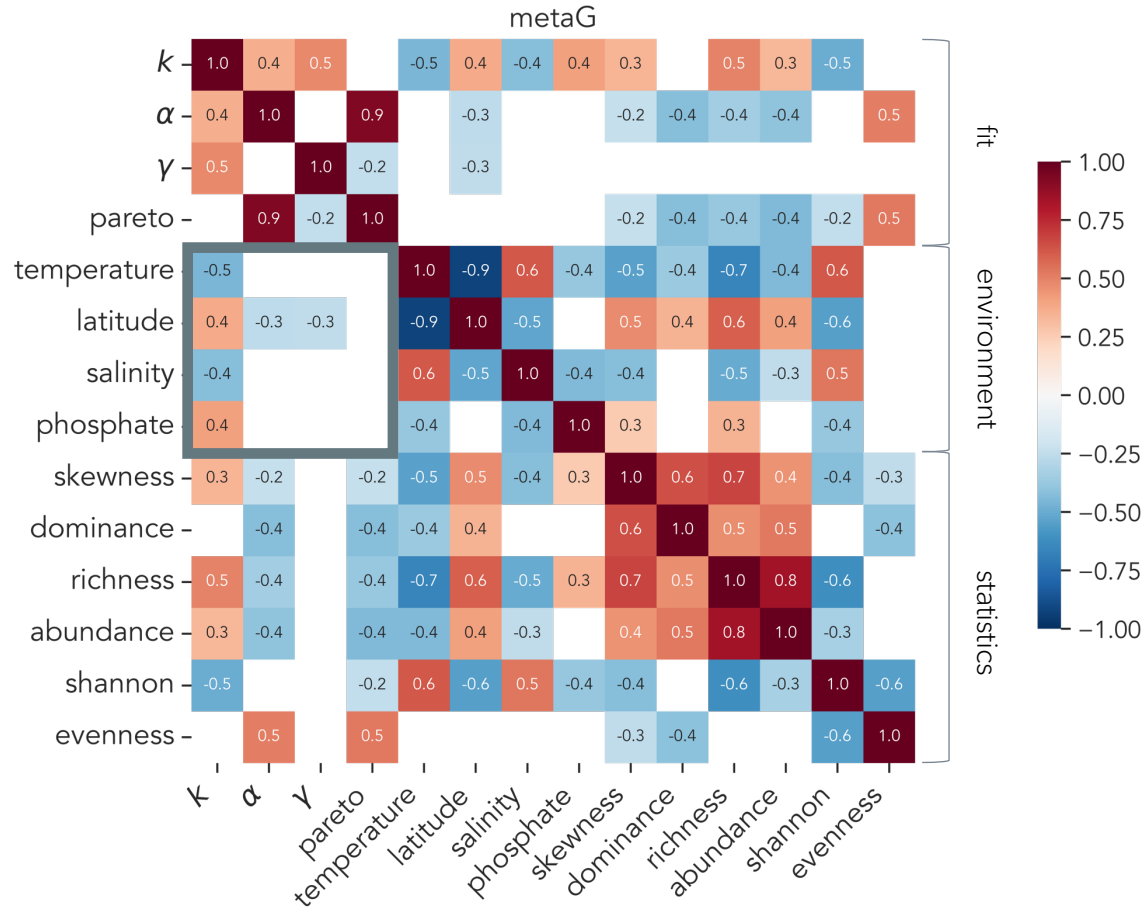
metaG exponents display **higher** and more **homogenous** exponents



# A relation between metaG and metaT?

$$P(x \gg k) \sim x^{-1-\frac{\alpha}{\gamma}}$$

$$P(x \ll k) \sim x^{-1+\frac{1}{\gamma}}$$



metaT exponents display more heterogeneity and regionality

metaG exponents display higher and more homogenous exponents

# An analytical framework for gene expression?

How to derive the Pareto distribution

PRL 97, 168302 (2006)

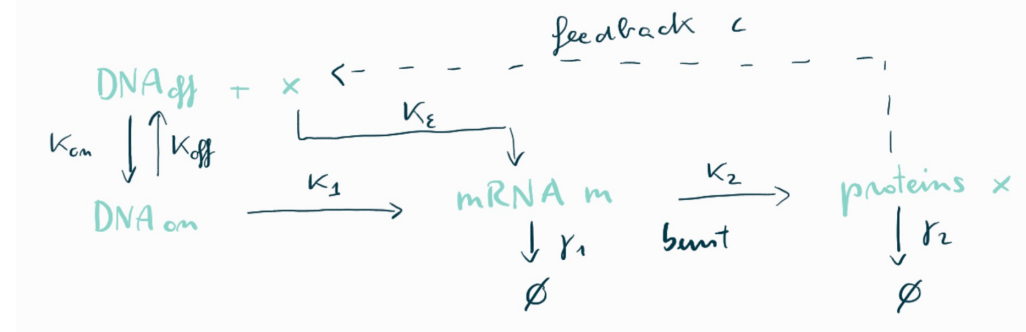
PHYSICAL REVIEW LETTERS

week ending  
20 OCTOBER 2006

## Linking Stochastic Dynamics to Population Distribution: An Analytical Framework of Gene Expression

Nir Friedman, Long Cai, and X. Sunney Xie

Department of Chemistry and Chemical Biology, Harvard University, 12 Oxford St., Cambridge, Massachusetts 02138, USA  
(Received 9 June 2006; published 19 October 2006)



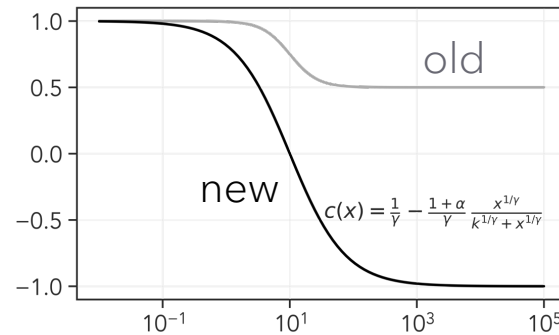
Removing bursts + slightly different feedback: Pareto IV distribution!

$$P(x) = \frac{\alpha}{\gamma k} \left[ 1 + \left( \frac{x}{k} \right)^{1/\gamma} \right]^{-1-\alpha} \left( \frac{x}{k} \right)^{-1+1/\gamma}$$



A biological explanation for the Pareto exponents...

But... we impose the feedback to be a negative function + the model is for proteins, not RNA... still working on it!



From the Master Equation

assuming burst and feedback (Hill function)

$$p(x) = Ax^{a(1+\varepsilon)-1} e^{-x/b} [1 + (x/k)^H]^{-a/H}$$

## Analytical distributions for detailed models of stochastic gene expression in eukaryotic cells

Zhixing Cao<sup>a,b</sup> and Ramon Grima<sup>b,1</sup>

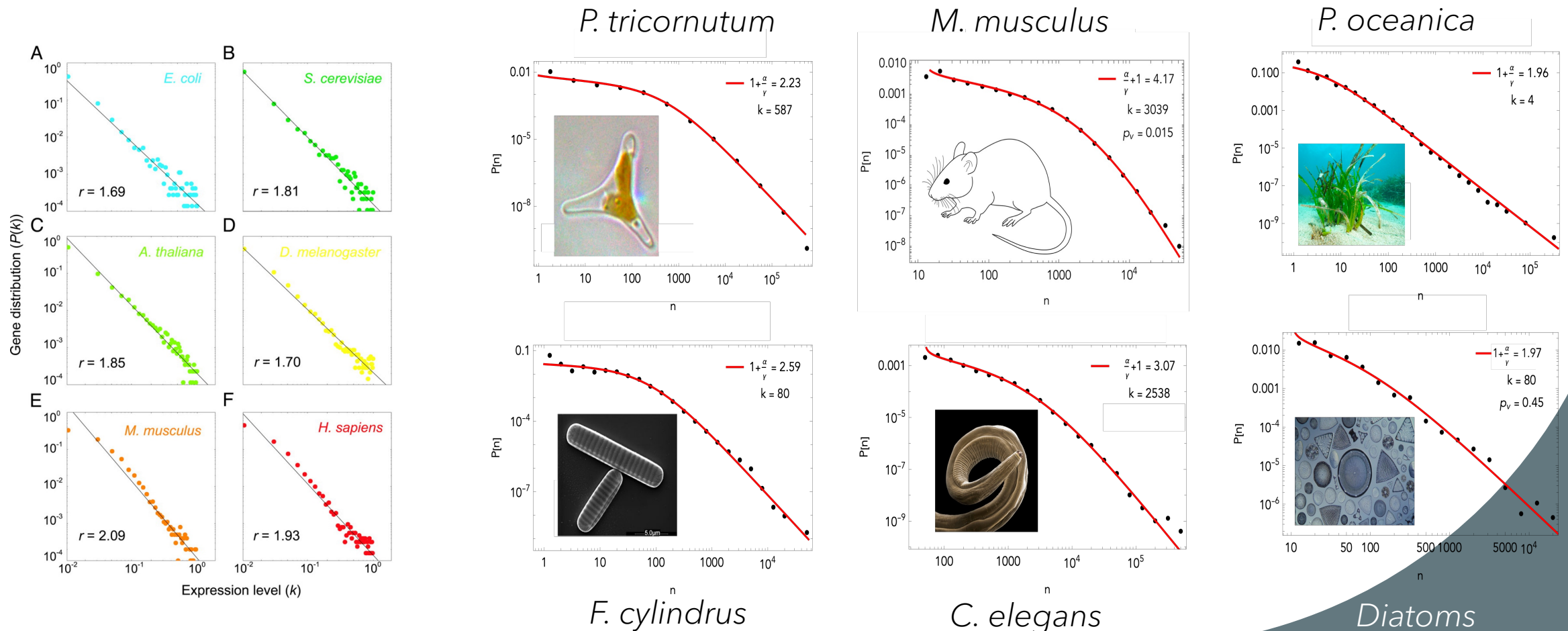


# "Universal" distribution for gene expression?

## Universality and flexibility in gene expression from bacteria to human

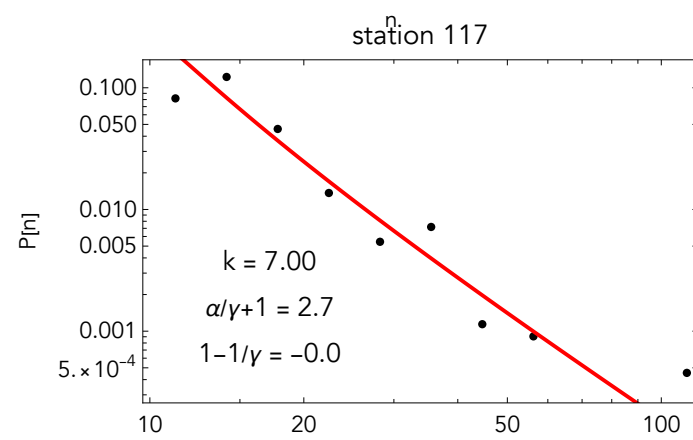
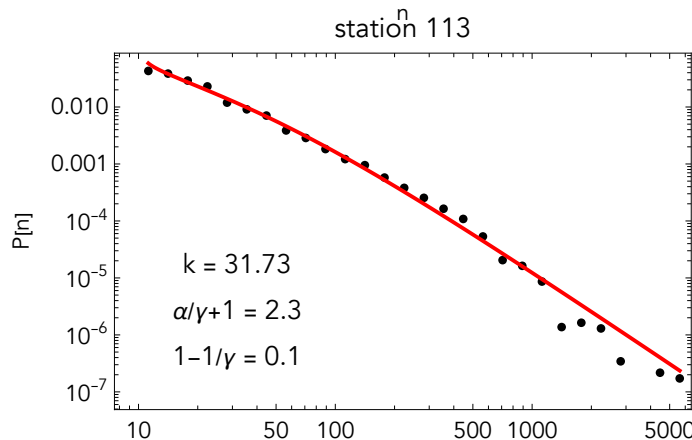
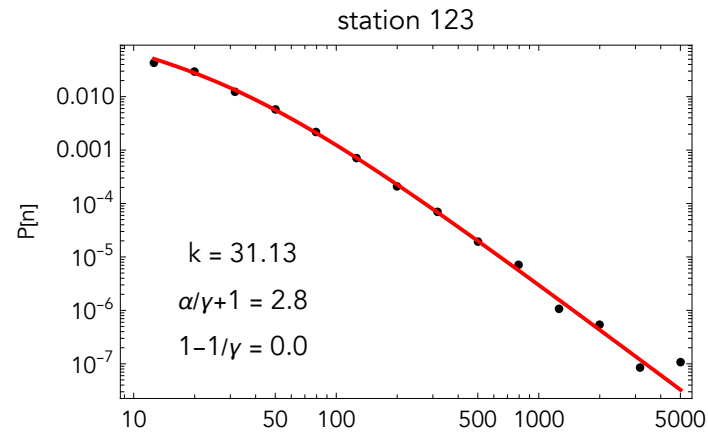
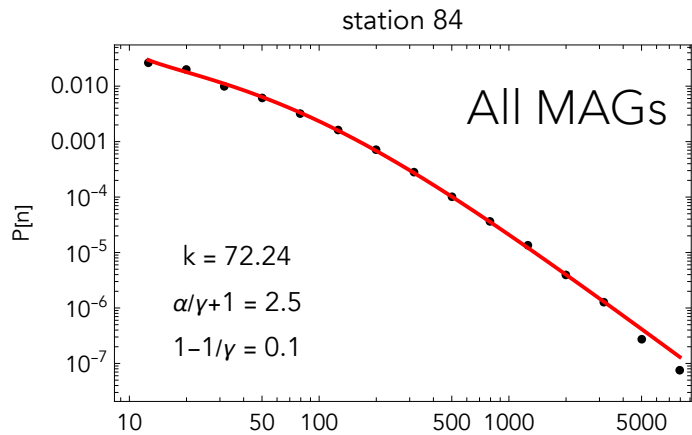
Hiroki R. Ueda, Satoko Hayashi, Shinichi Matsuyama, Tetsuya Yomo, Seiichi Hashimoto, Steve A. Kay, John B. Hogenesch, and Masamitsu Iino [-4](#) [Authors Info & Affiliations](#)

March 3, 2004 | 101 (11) 3765-3769 | <https://doi.org/10.1073/pnas.0306244101>

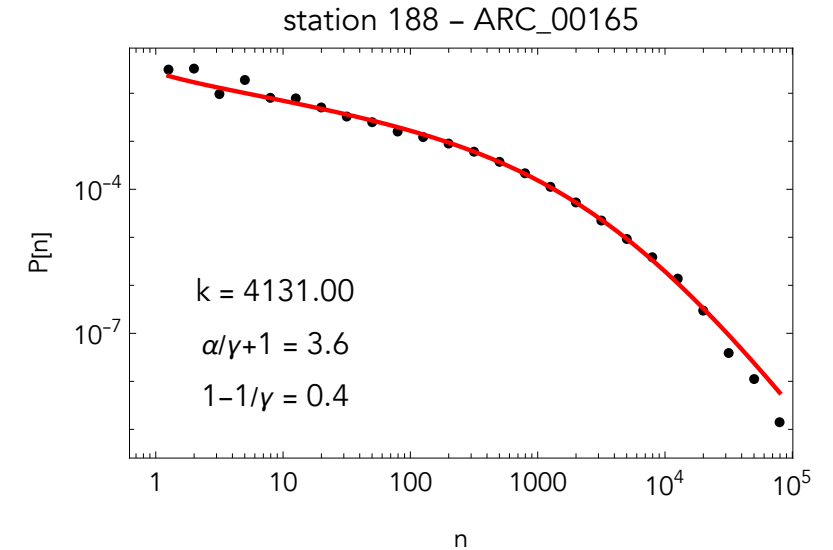


# "Universal" distribution for gene expression?

Looking at MAGs

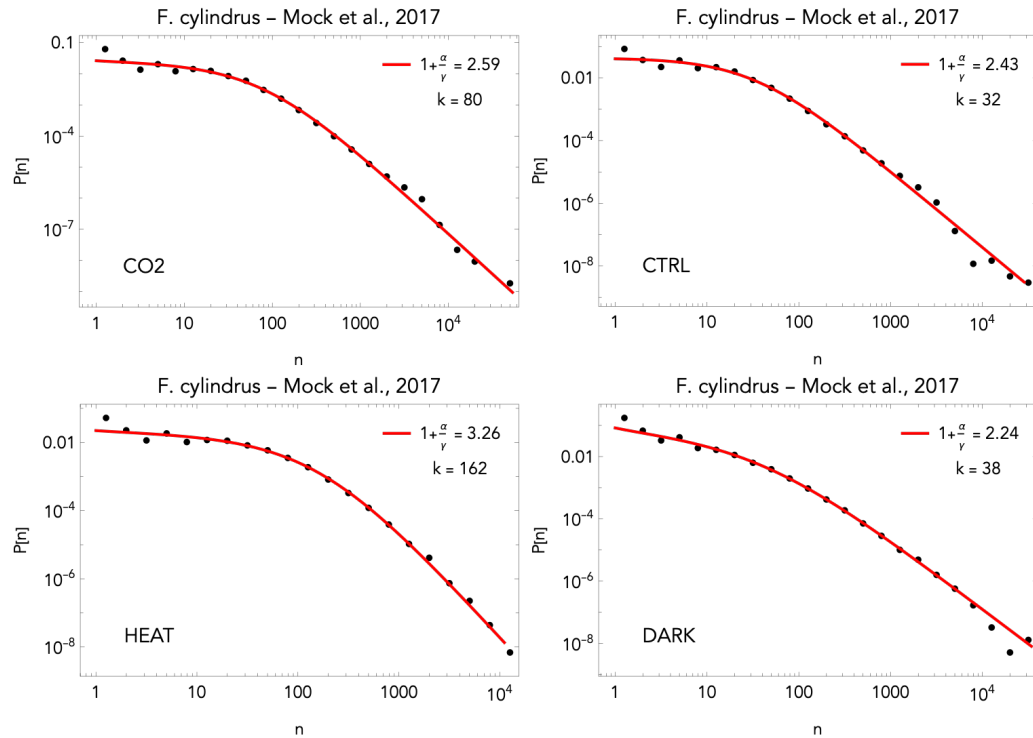


Focusing on only 1 MAG

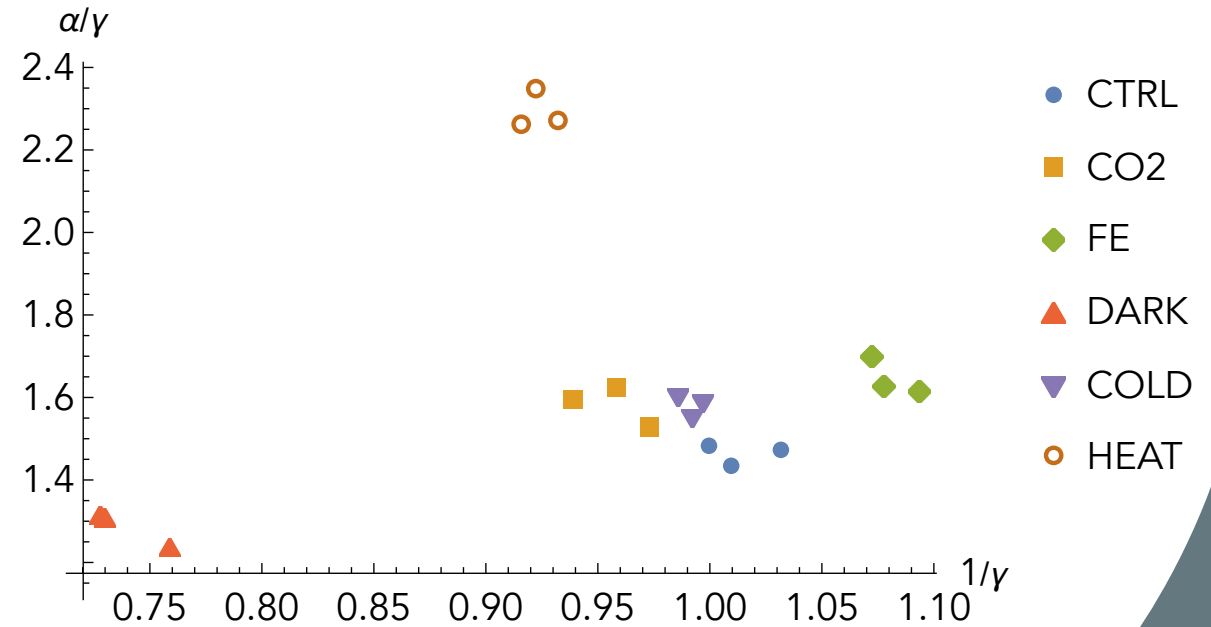


It usually works, when the MAGs are enough...

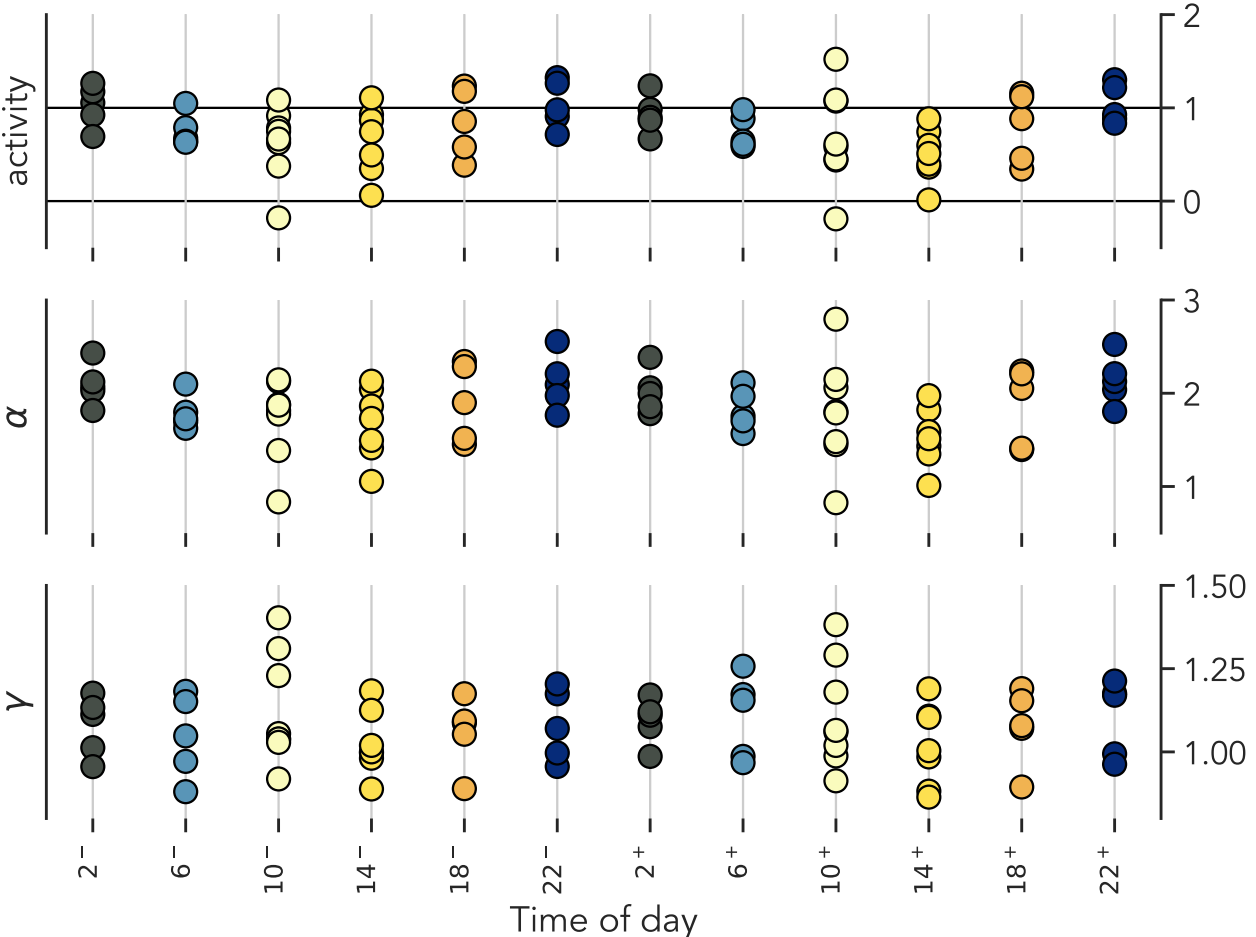
# Assessing the role of biology



*F. cylindrus* – Mock et al., 2017



# Assessing the role of biology



**PLOS GENETICS**

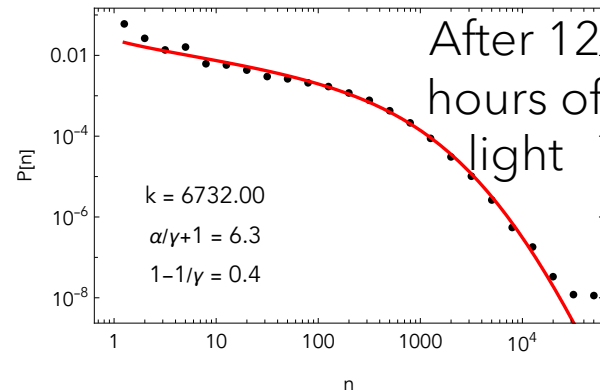
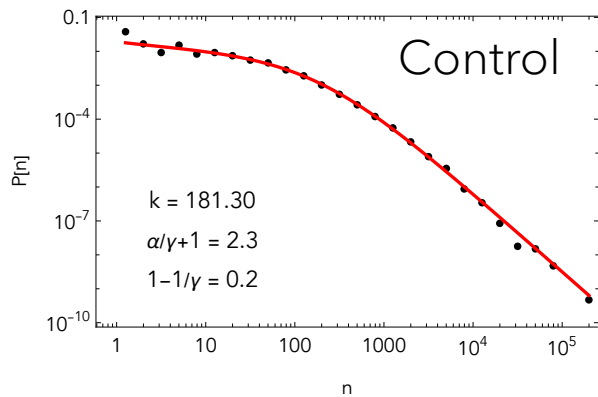
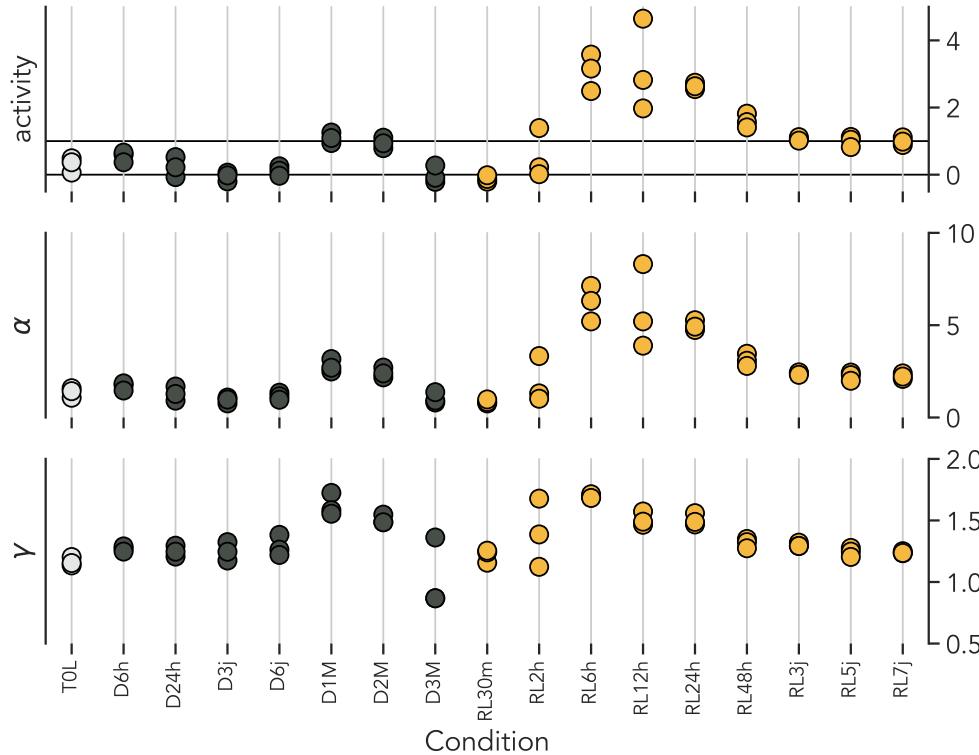
**Transcriptional Orchestration of the Global Cellular Response of a Model Pennate Diatom to Diel Light Cycling under Iron Limitation**

Sarah R. Smith, Jeroen T. F. Gillard, Adam B. Kustka, John P. McCrow, Jonathan H. Badger, Hong Zheng, Ashley M. New, Chris L. Dupont, Toshihiro Obata, Alisdair R. Fernie, Andrew E. Allen

We can derive the mean activity per cell from the exponents of the Pareto IV!

$$\text{activity} = \frac{1}{\gamma} + \frac{\alpha}{\gamma}$$

# Assessing the role of biology

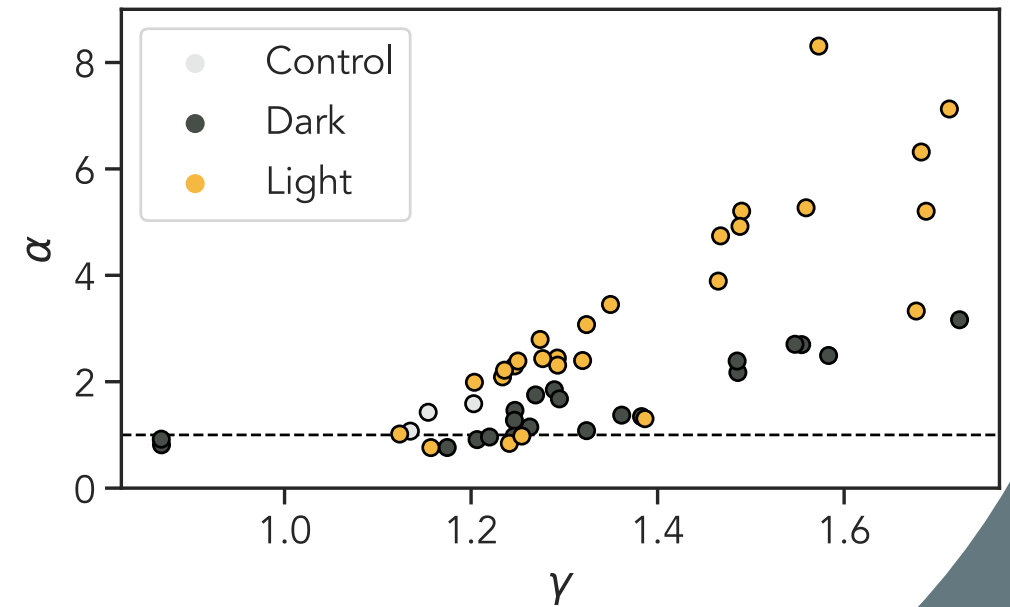


Open Access | Published: 16 January 2017

## Evolutionary genomics of the cold-adapted diatom *Fragilariopsis cylindrus*

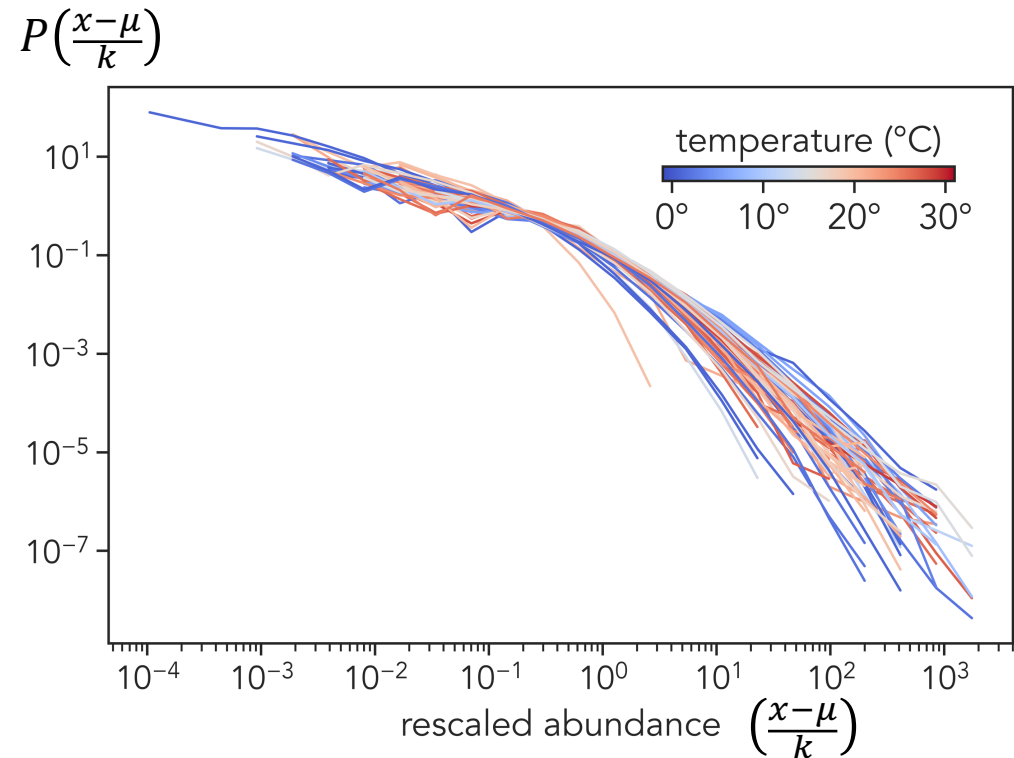
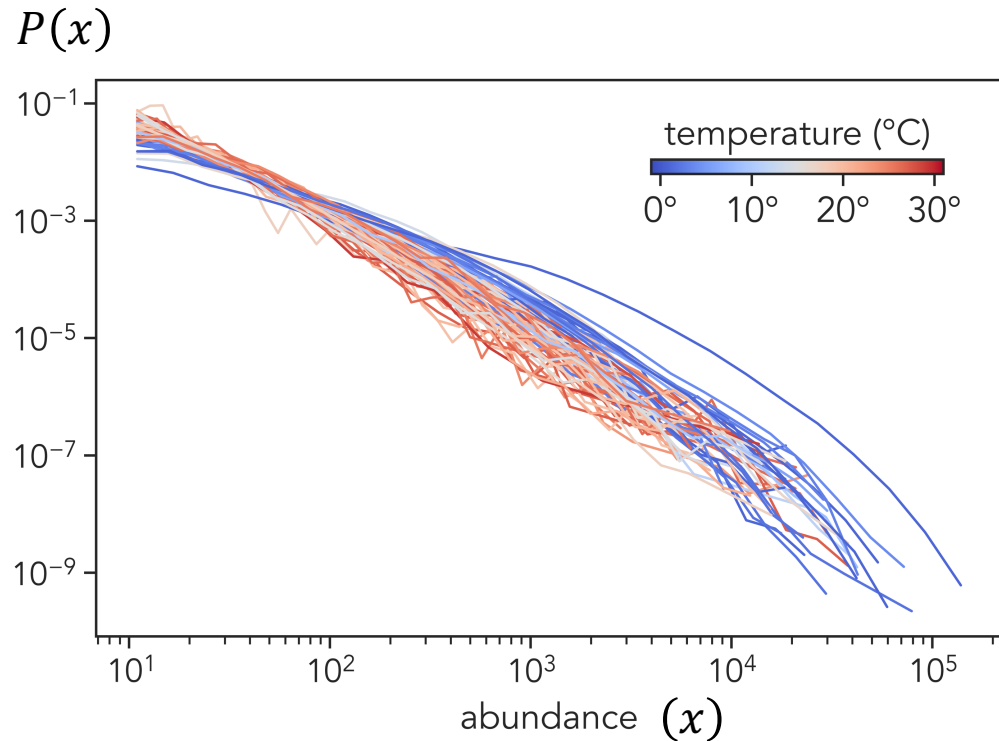
Thomas Mock , Robert P. Otilar, Jan Strauss, Mark McMullan, Pirita Paajanen, Jeremy Schmutz, Asaf Salamov, Remo Sanges, Andrew Toseland, Ben J. Ward, Andrew E. Allen, Christopher L. Dupont, Stephan Frickenhaus, Florian Maumus, Alaguraj Veluchamy, Taoyang Wu, Kerrie W. Barry, Angela Falciatore, Maria I. Ferrante, Antonio E. Fortunato, Gernot Glöckner, Ansgar Gruber, Rachel Hipkin, Michael G. Janech, ... Igor V. Grigoriev Show authors

*Nature* 541, 536–540 (2017) | [Cite this article](#)



$$\text{activity} = \frac{1}{\gamma} + \frac{\alpha}{\gamma}$$

# SAD normalization?



Pareto IV distribution:

$$P(x|k, \alpha, \gamma, \mu) = \frac{\alpha}{\gamma k} \left[ 1 + \left( \frac{x-\mu}{k} \right)^{1/\gamma} \right]^{-1-\alpha} \left( \frac{x-\mu}{k} \right)^{-1+1/\gamma}$$



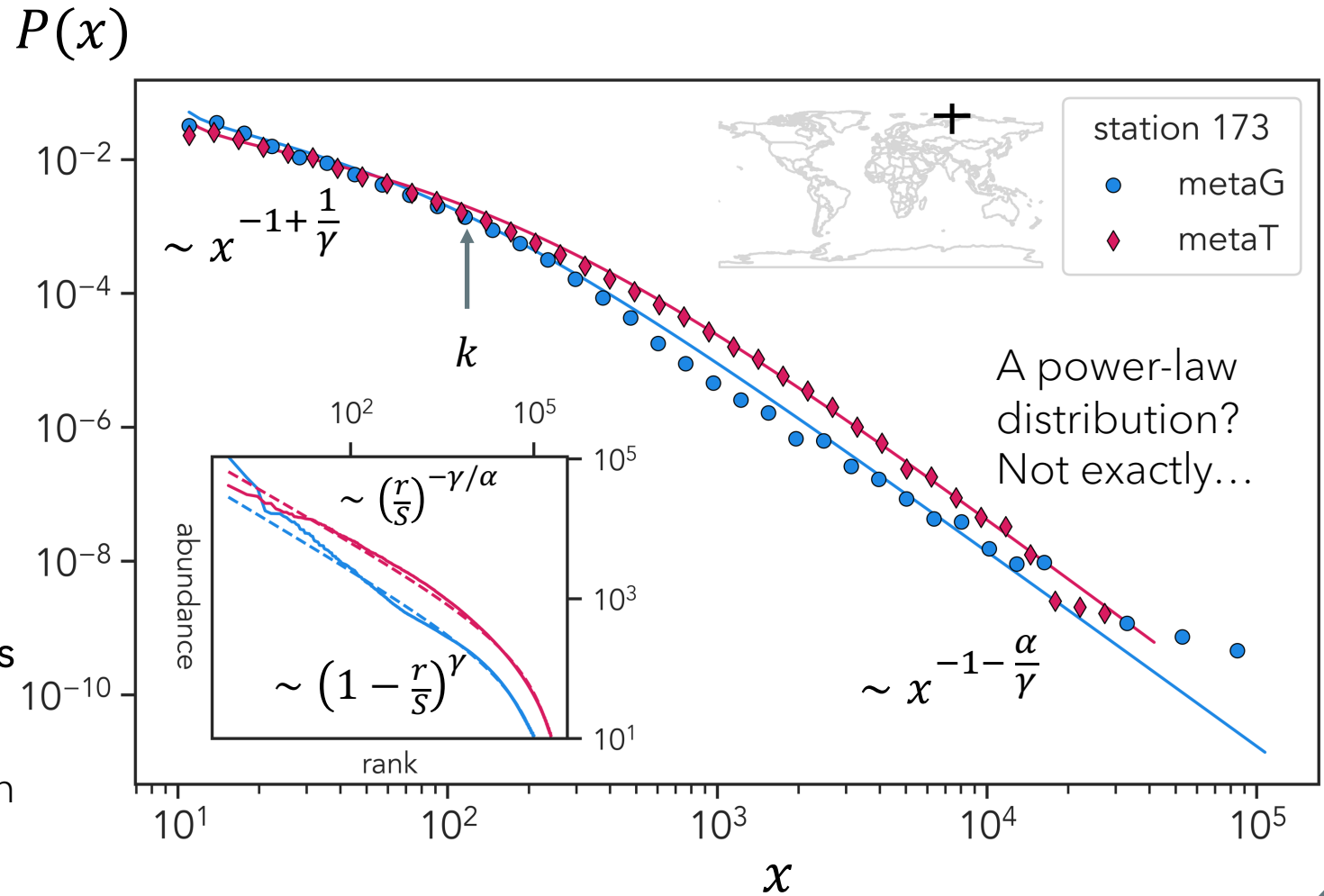
# SAD & RAD

Open questions:

- Is there a relation between metaG and metaT? **Probably no**
- Why the Pareto IV? Is there a generative model? **Probably yes**
- What is the biological meaning of the parameters? **Yes, model**
- Does it work also for transcriptomics data? **Yes**
- Do we have to take it into account in the data normalization? **Probably**

Pareto IV distribution:

$$P(x|k, \alpha, \gamma, \mu) = \frac{\alpha}{\gamma k} \left[ 1 + \left( \frac{x-\mu}{k} \right)^{1/\gamma} \right]^{-1-\alpha} \left( \frac{x-\mu}{k} \right)^{-1+1/\gamma}$$



- Analytical formula for the Rank Abundance distribution:  $RAD(r) = \mu + k \left[ \left(\frac{r}{s}\right)^{-1/\alpha} - 1 \right]^\gamma$

# Thank you!



Daniele Iudicone



Samir Suweis



Emanuele Pigani



Maurizio  
Ribera d'Alcalà



Sandro Azaele



Lucia Campese



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LIPh Lab!

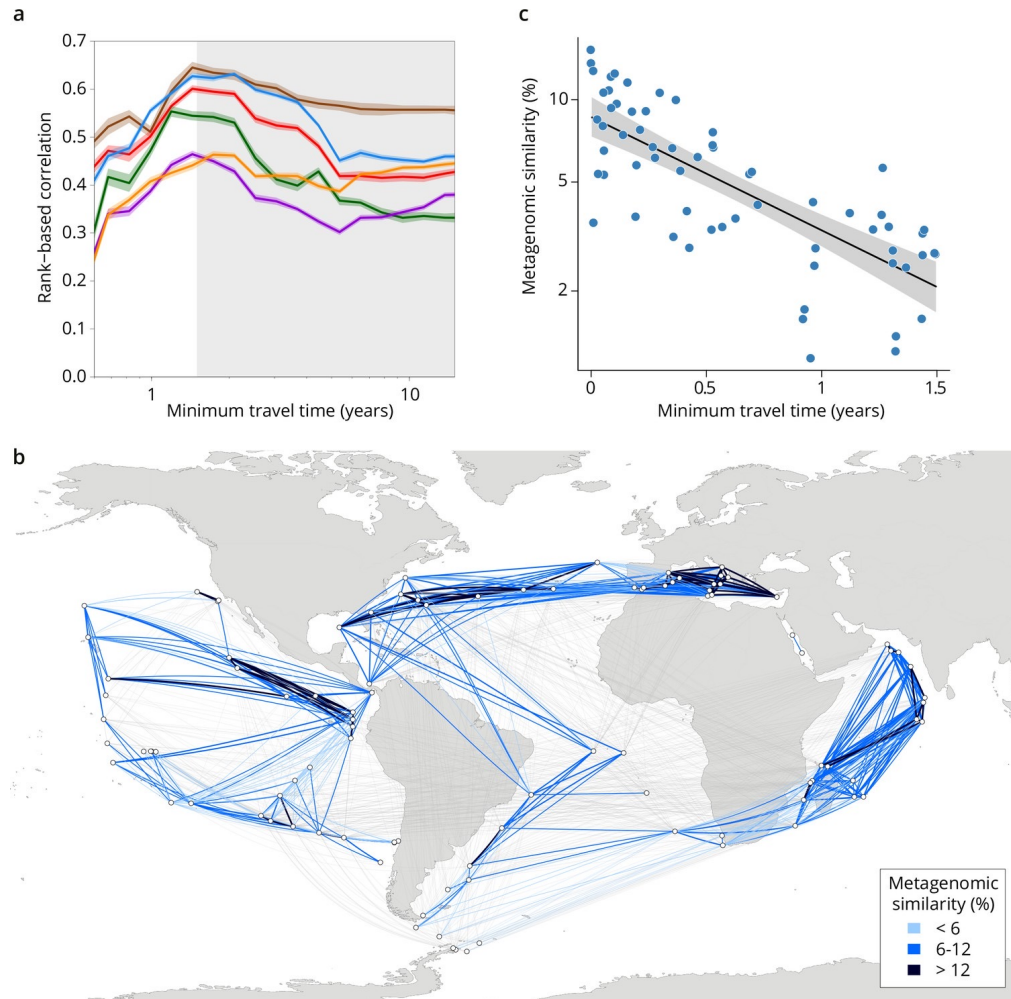
## Time for the ring model?

# Genomic evidence for global ocean plankton biogeography shaped by large-scale current systems

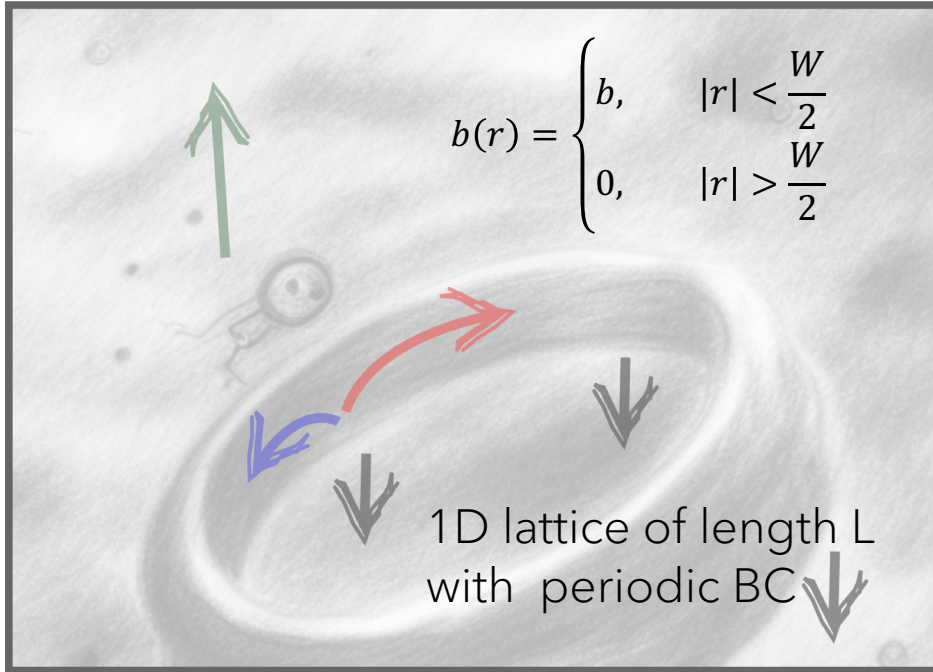
Daniel J Richter, Romain Watteaux, Thomas Vannier, Jade Leconte, Paul Frémont, Gabriel Reygondeau, Nicolas Maillet, Nicolas Henry, Gaëtan Benoit, Ophélie Da Silva, Tom O Delmont, Antonio Fernández-Guerra, Samir Suweis, Romain Narcis, Cédric Berney, Damien Eveillard, Frederick Gavory, Lionel Guidi, Karine Labadie, Eric Mahieu, Julie Poulain, Sarah Romac, Simon Roux, Céline Dimier, Stefanie Kandels, Marc Picheral, Sarah Searson, Tara Oceans Coordinators, Stéphane Pesant, Jean-Marc Aury, Jennifer R Brum, Claire Lemaitre, Eric Pelletier, Peer Bork, Shinichi Sunagawa, Fabien Lombard, Lee Karp-Boss, Chris Bowler, Matthew B Sullivan, Eric Karsenti, Mahendra Mariadassou, Ian Probert, Pierre Peterlongo, Patrick Wincker, Colombran de Vargas, Maurizio Ribera d'Alcalá, Daniele Iudicone, Olivier Jaillon [« see less](#)

The metagenomic similarity decay exponentially with travel time (Lagrangian distance)

Why?



# The Ring model



$$b(r) = \begin{cases} b, & |r| < \frac{W}{2} \\ 0, & |r| > \frac{W}{2} \end{cases}$$

$$X_i \xrightarrow{b(r)} X_i + X_i$$

$$X_i \xrightarrow{d} \emptyset$$

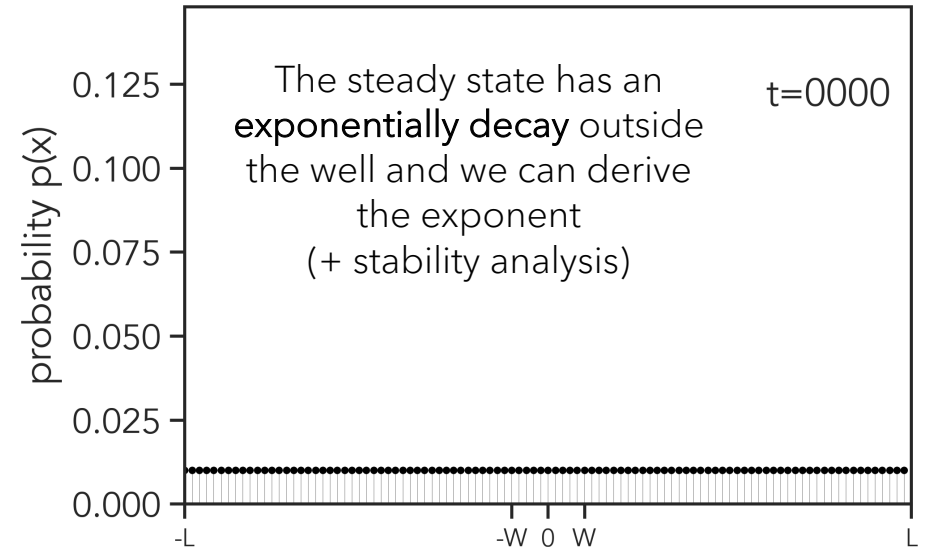
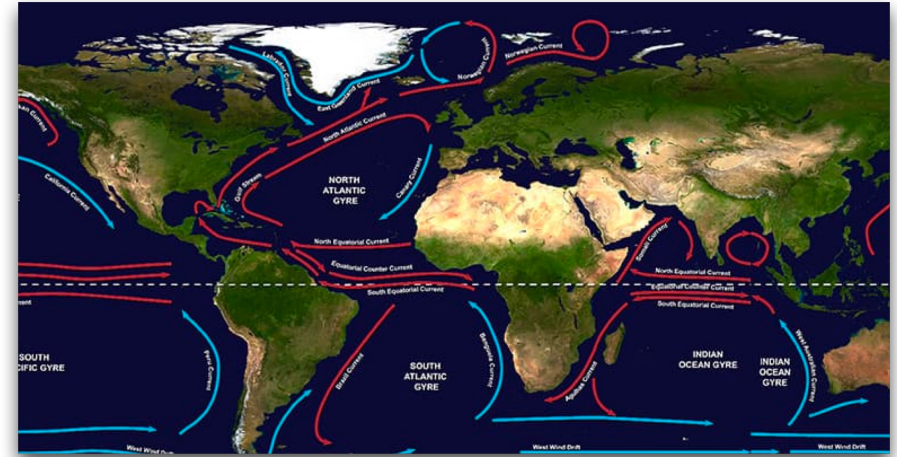
$$X_i \xrightarrow{p_+} X_{i+1}$$

$$X_i \xrightarrow{p_-} X_{i-1}$$

Continuum limit of the deterministic part of the Langevin equation:

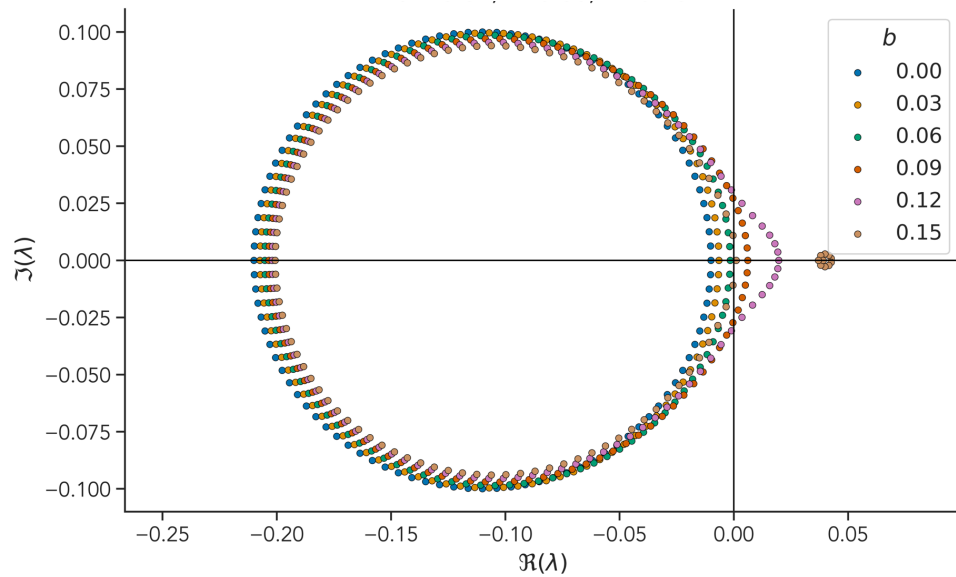
$$\frac{\partial x}{\partial t} = D \frac{\partial^2 x}{\partial r^2} - v \frac{\partial x}{\partial r} + \mu(r)x \quad (\sim \text{particle in a potential well})$$

$$D = p_+ ; v = p_+ - p_- ; \mu(r) = b(r) - d$$



By means of the stochastic terms we want to analytically investigate the decay in similarity...

# The Ring model

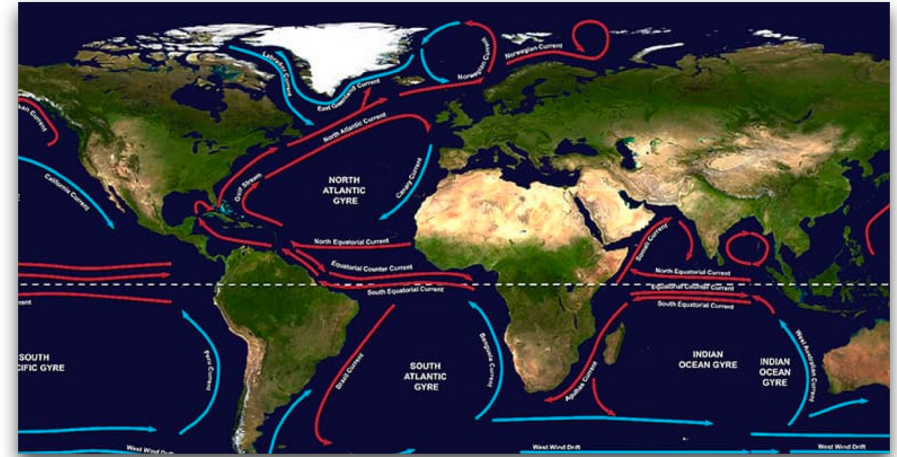


$$X_i \xrightarrow{b(r)} X_i + X_i$$

$$X_i \xrightarrow{d} \emptyset$$

$$X_i \xrightarrow{p_+} X_{i+1}$$

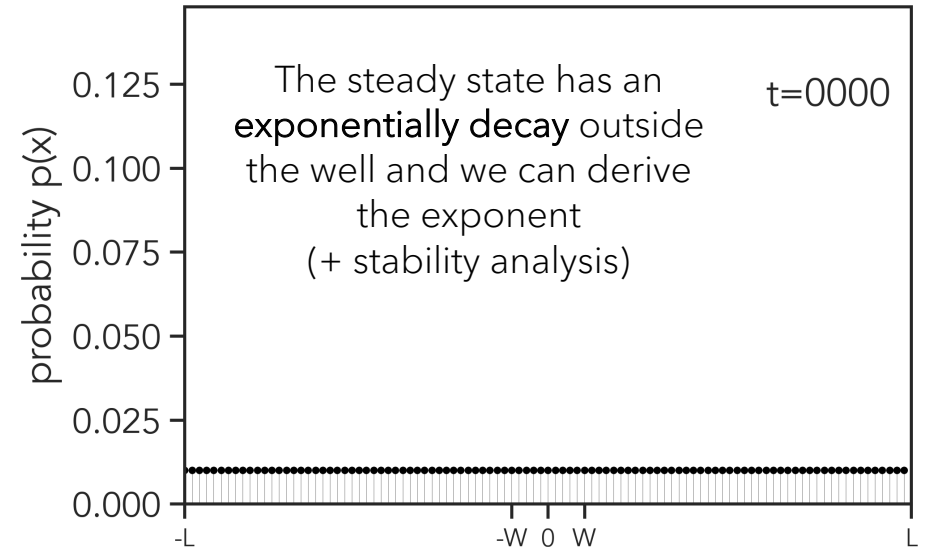
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By means of the stochastic terms we want to analytically investigate the decay in similarity...

# Thank you again!



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Samir Suweis



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Lucia Campese

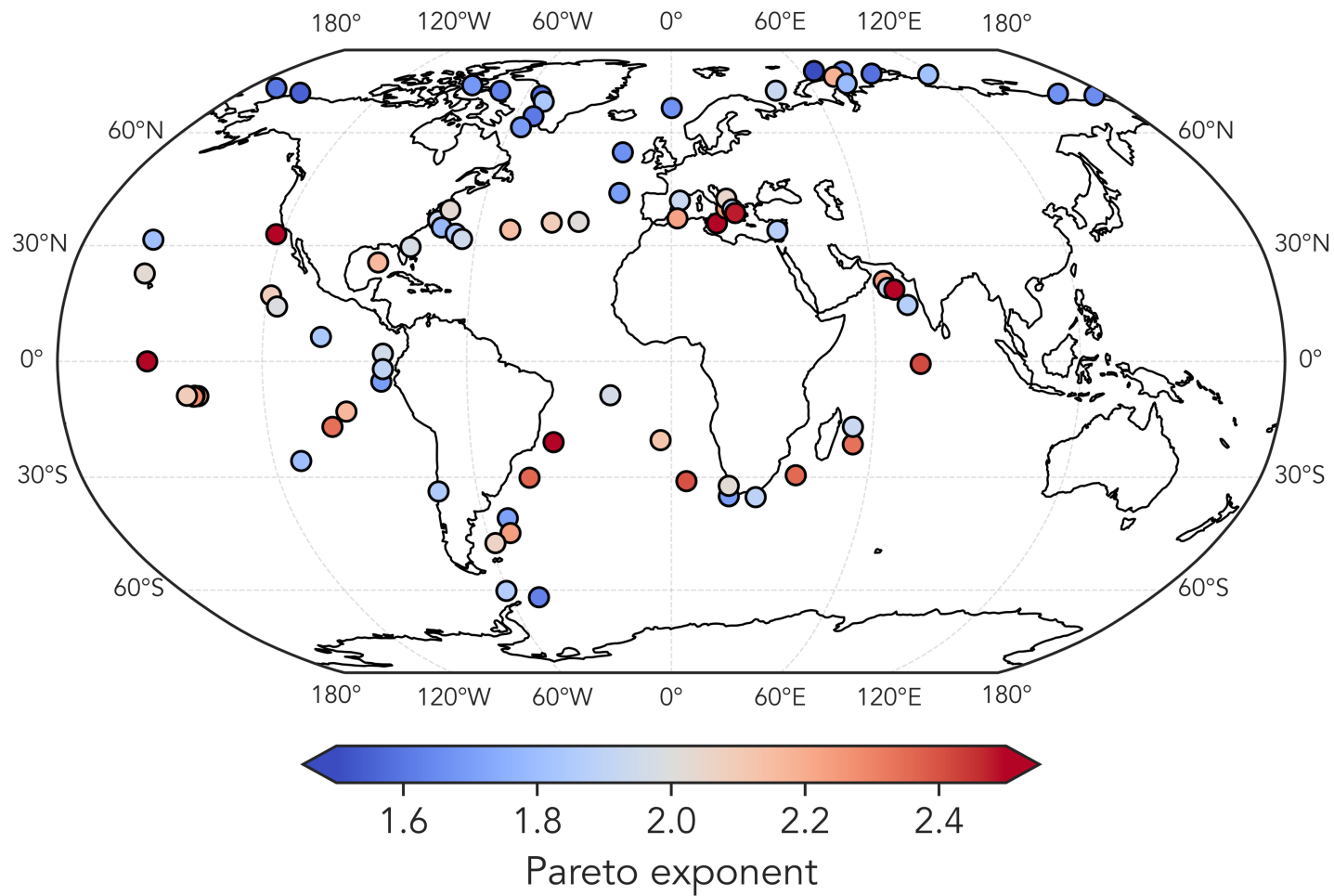
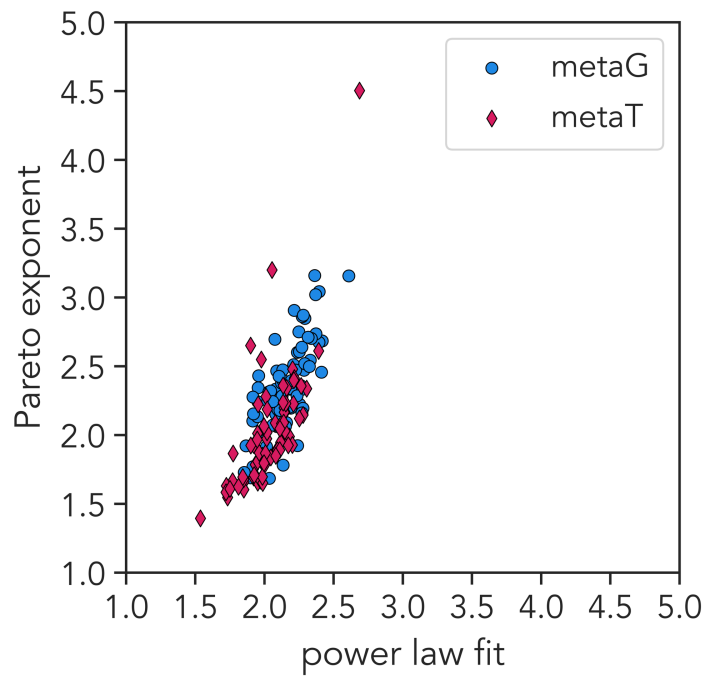
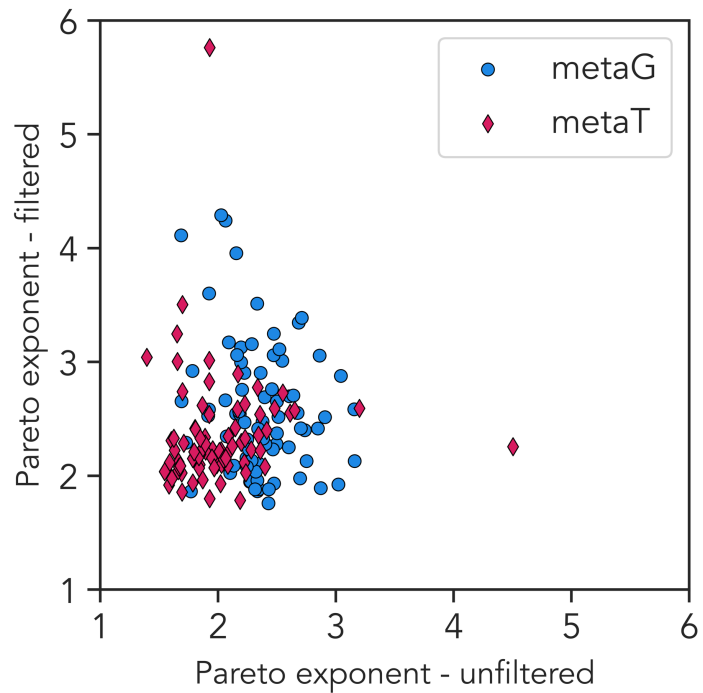


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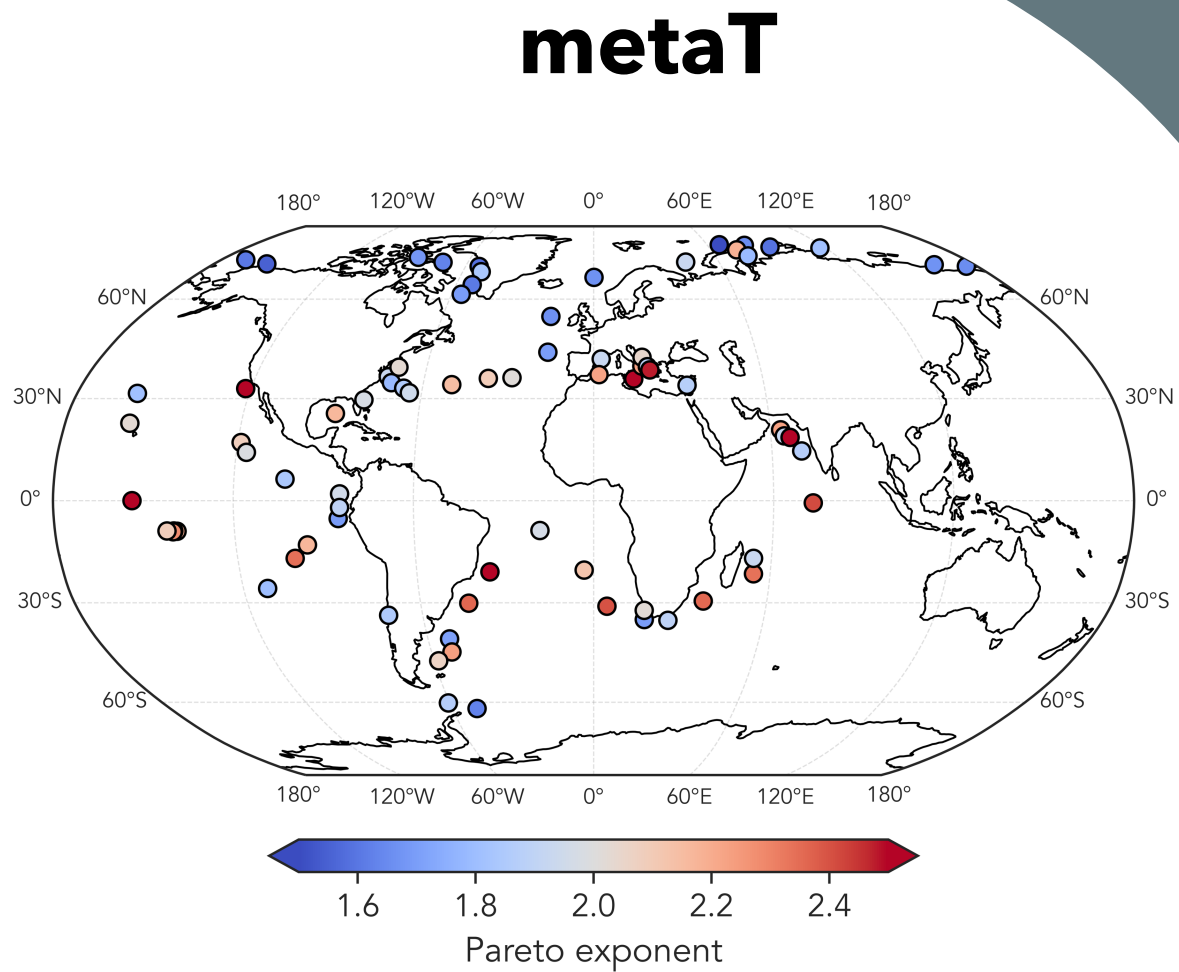
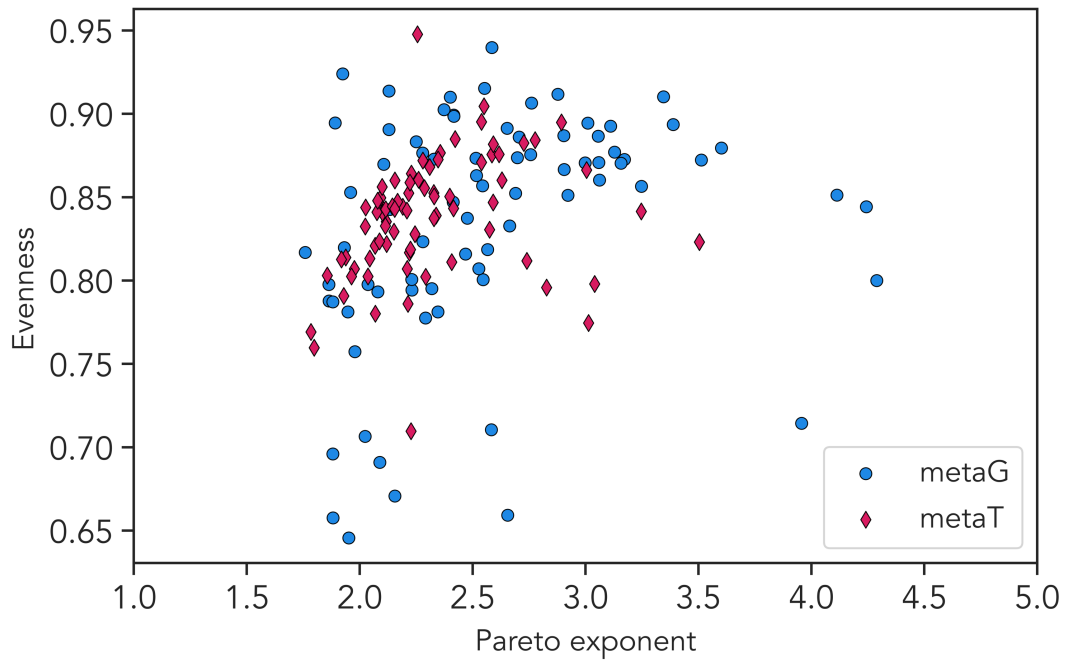
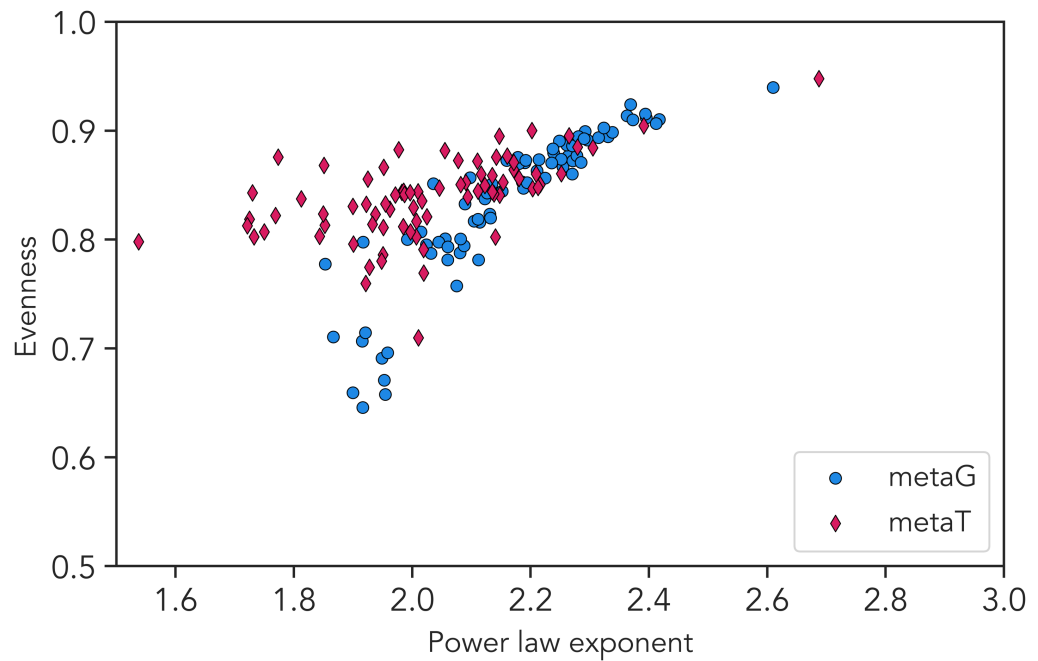
**That's all!**

# **BACKUP SLIDES**

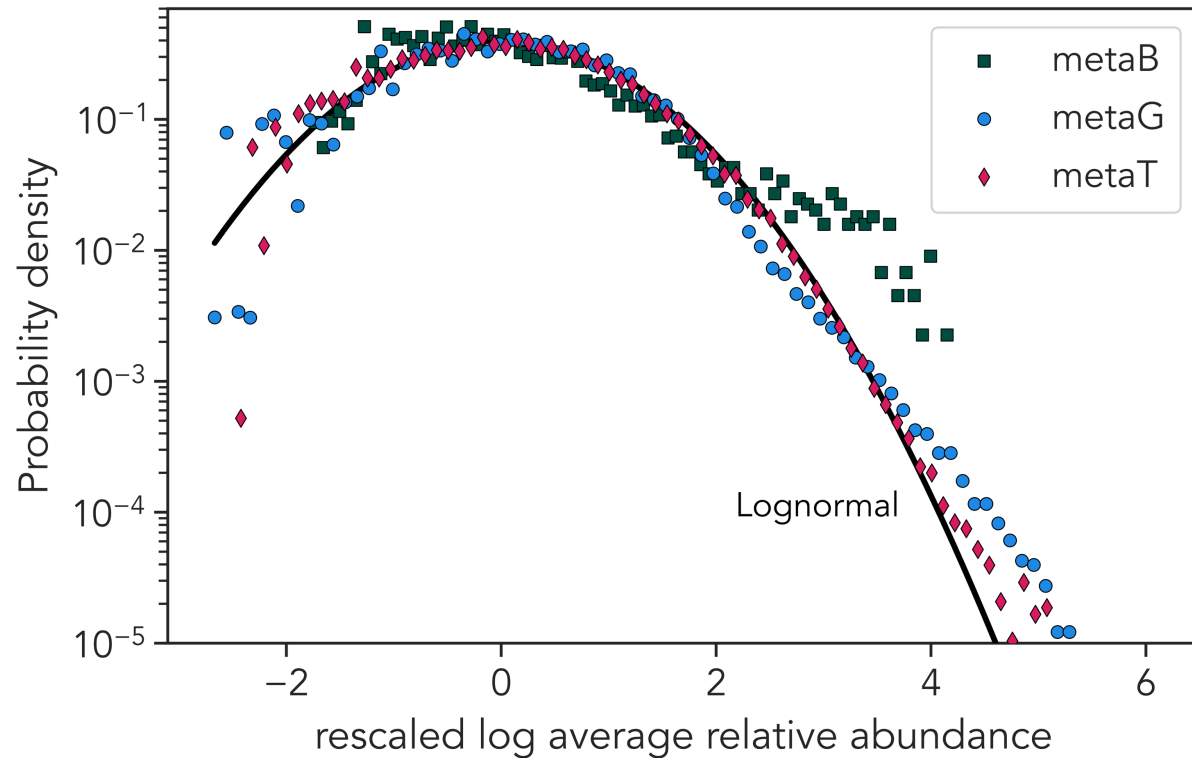
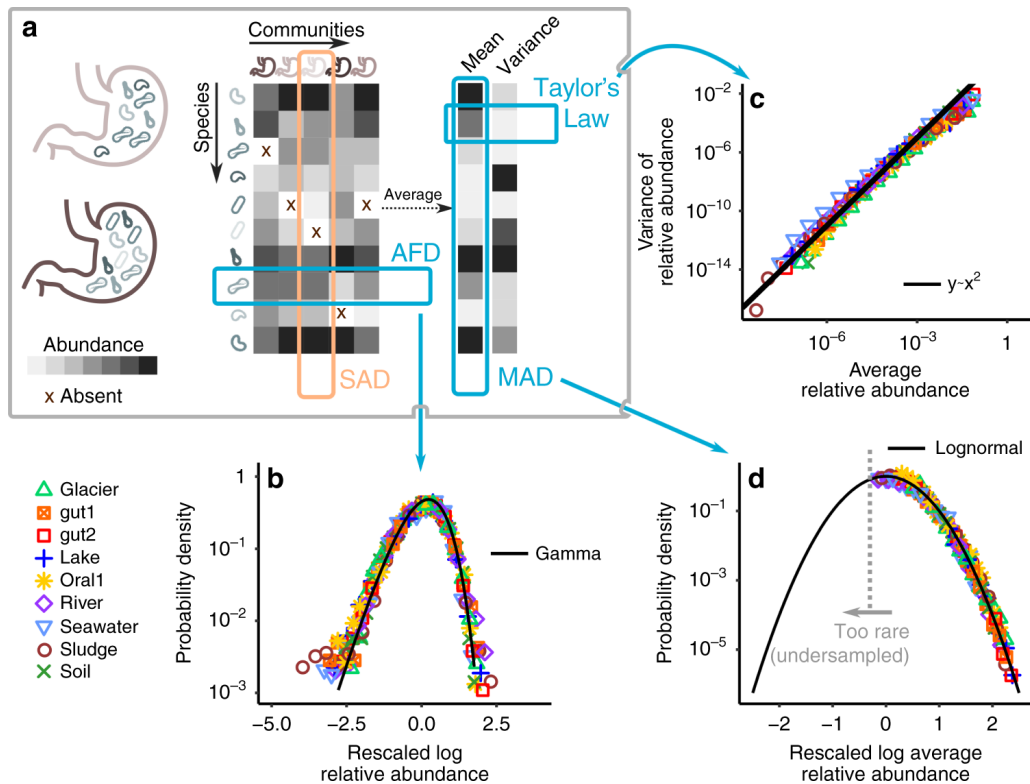
# metaT



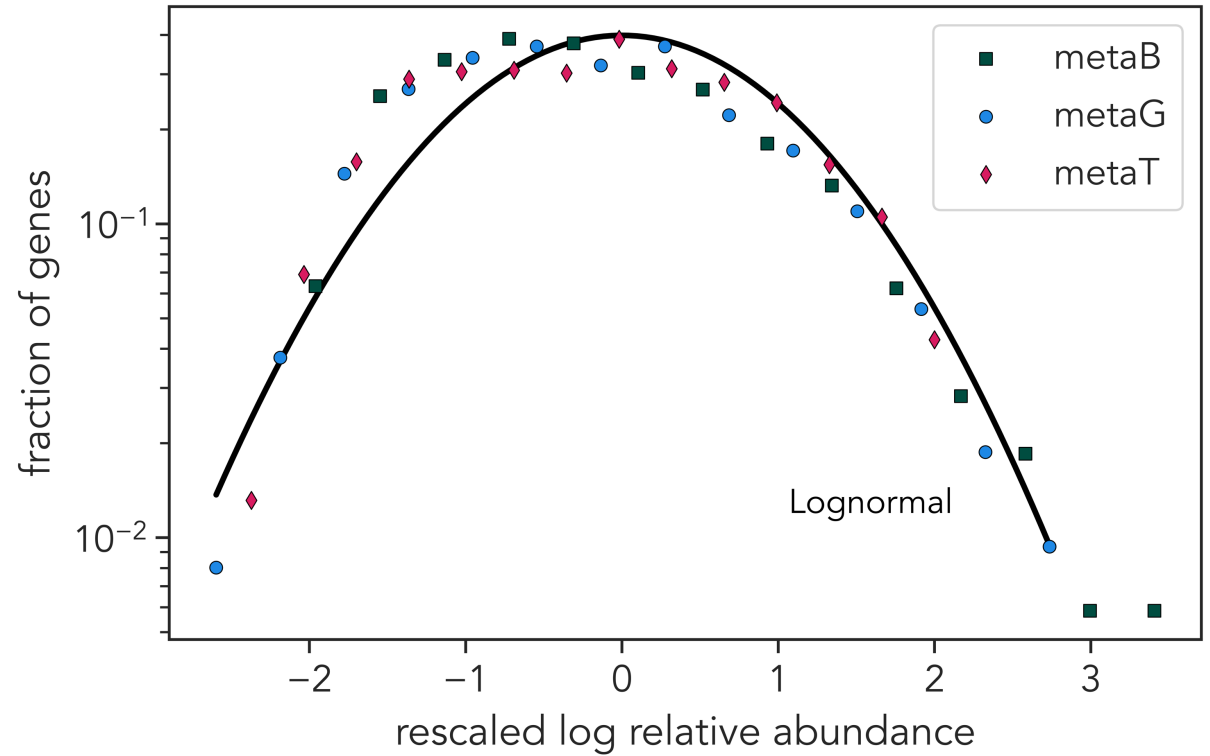
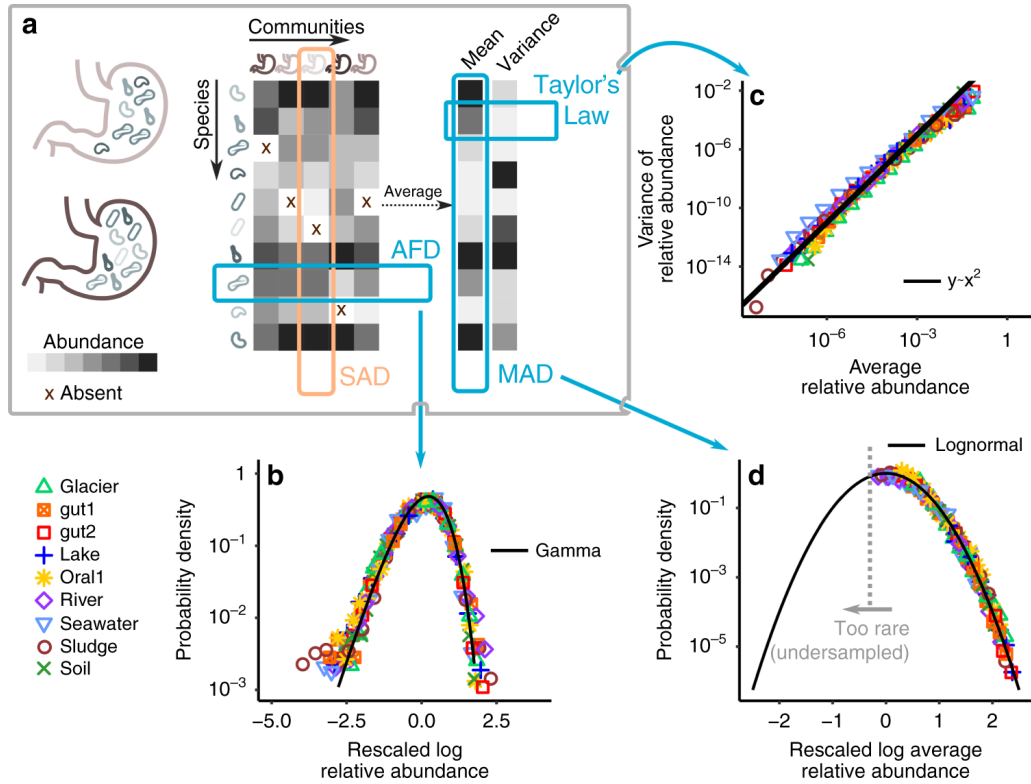




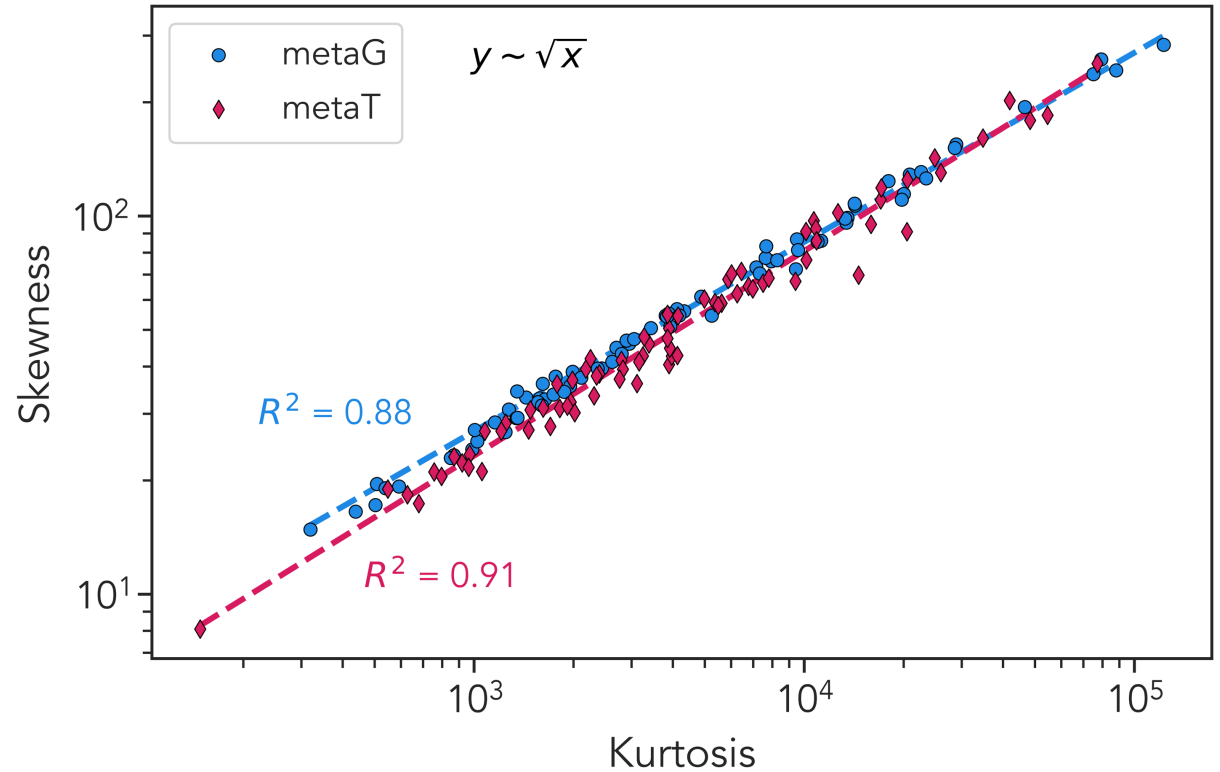
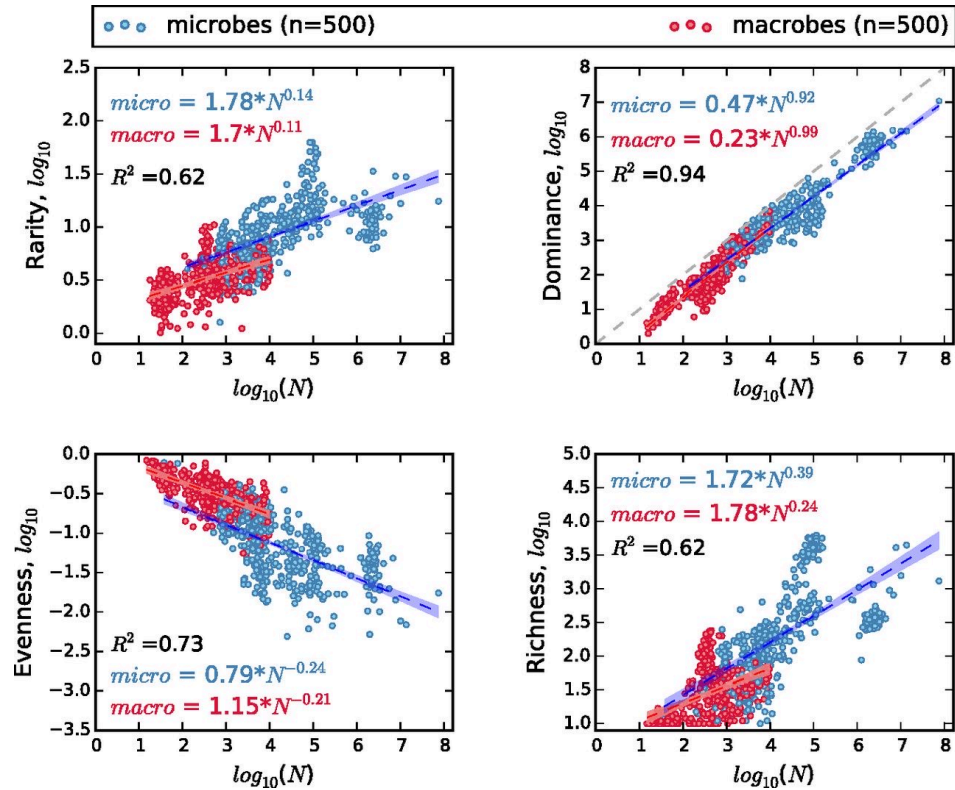
# MAD



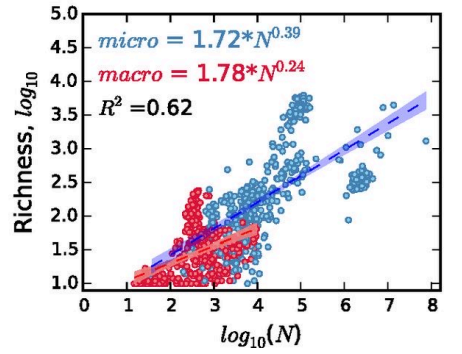
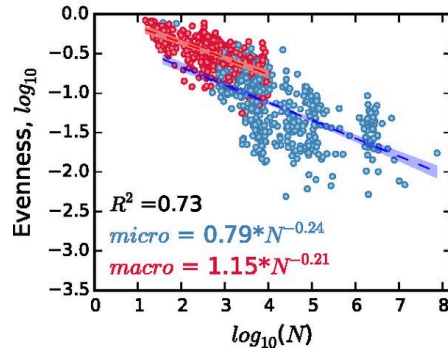
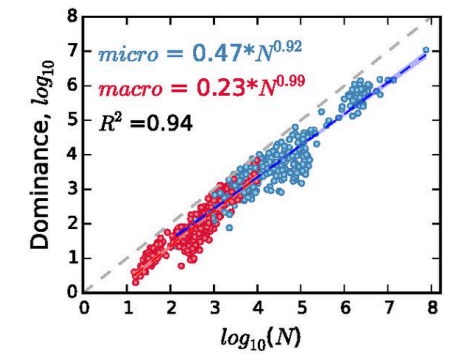
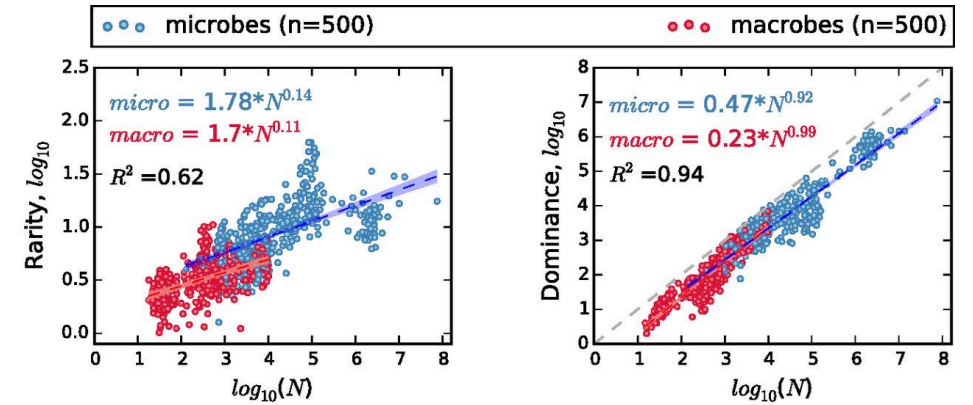
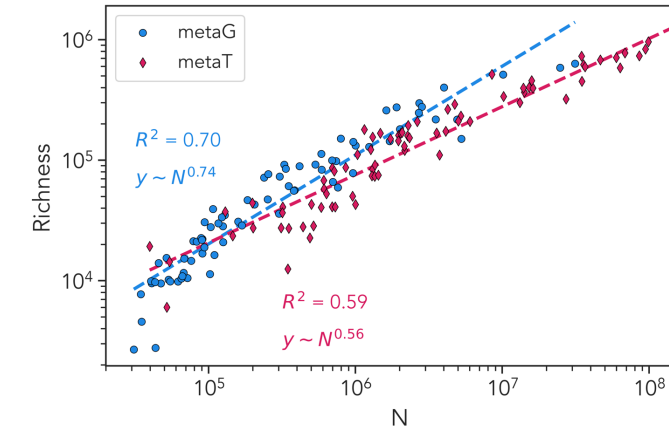
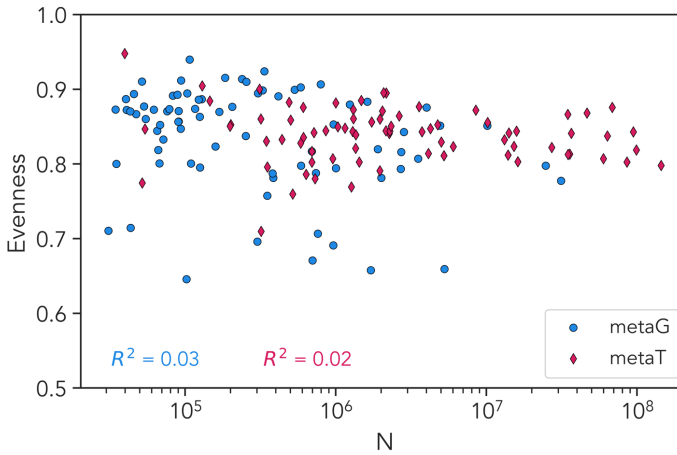
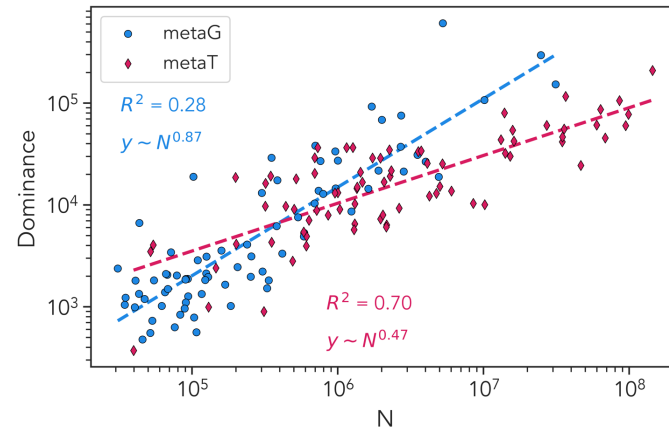
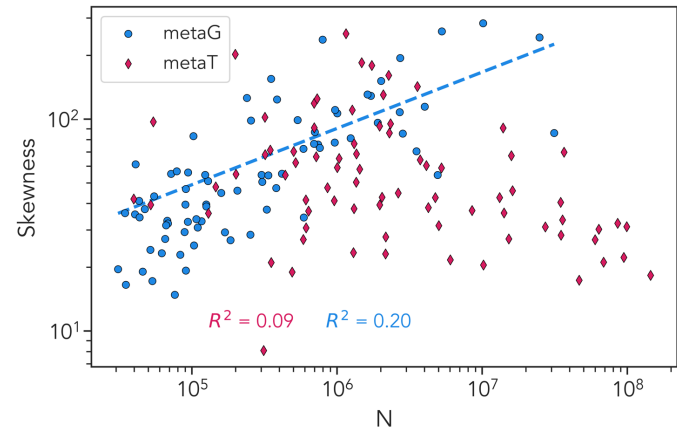
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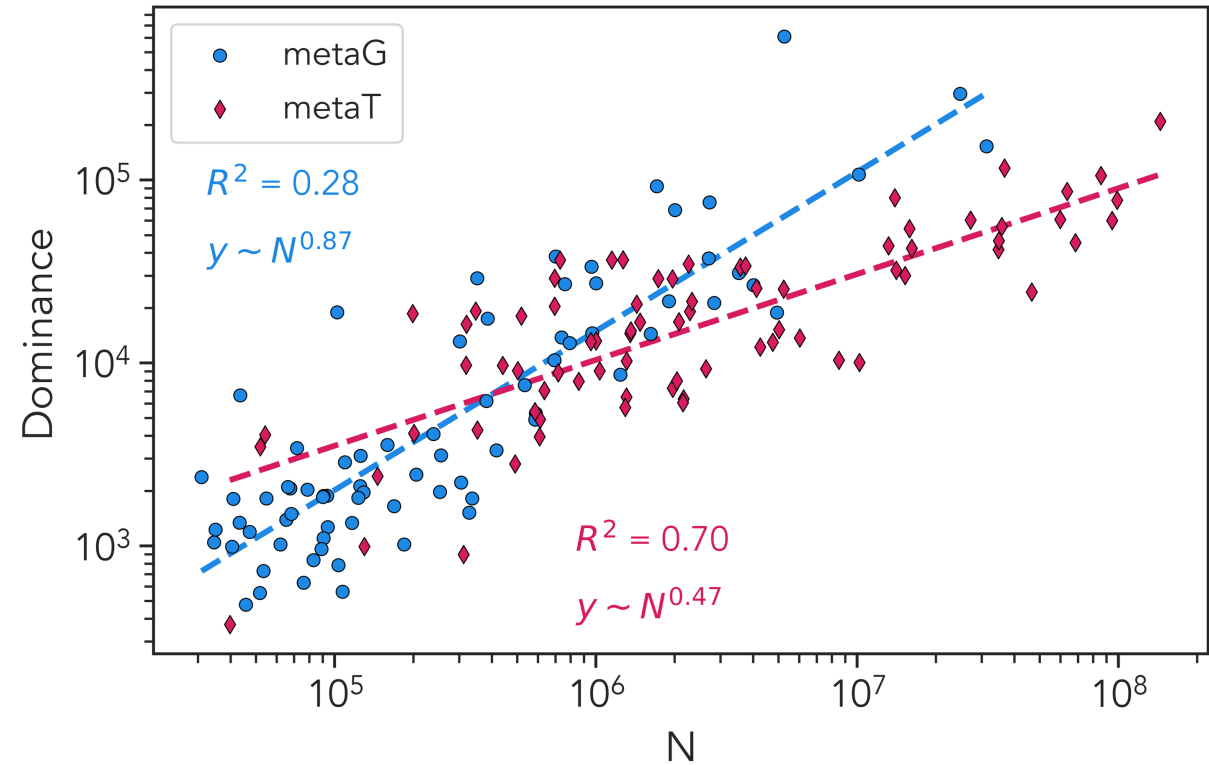
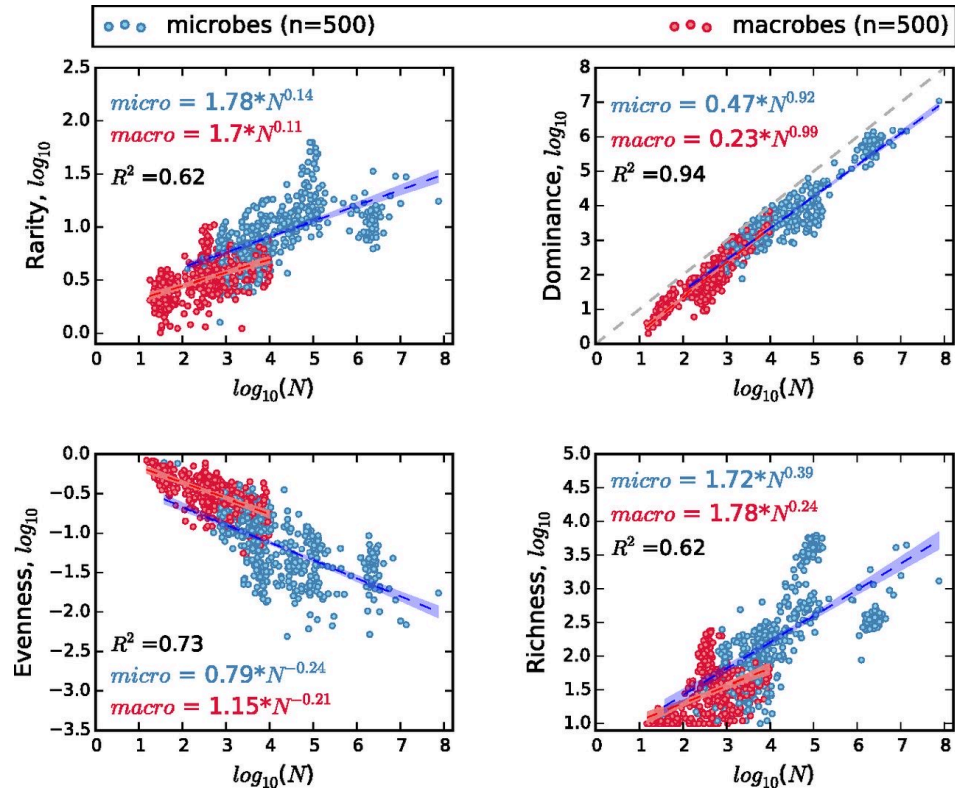
# Macroecological patterns



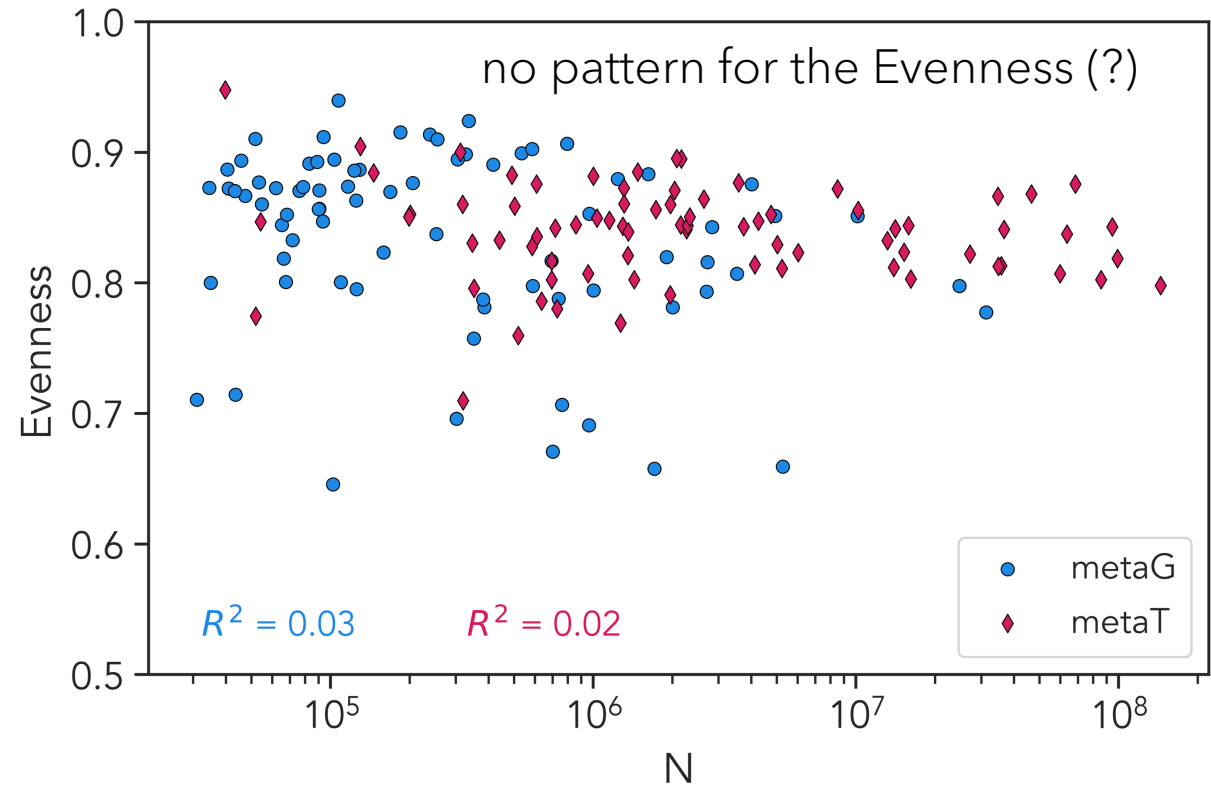
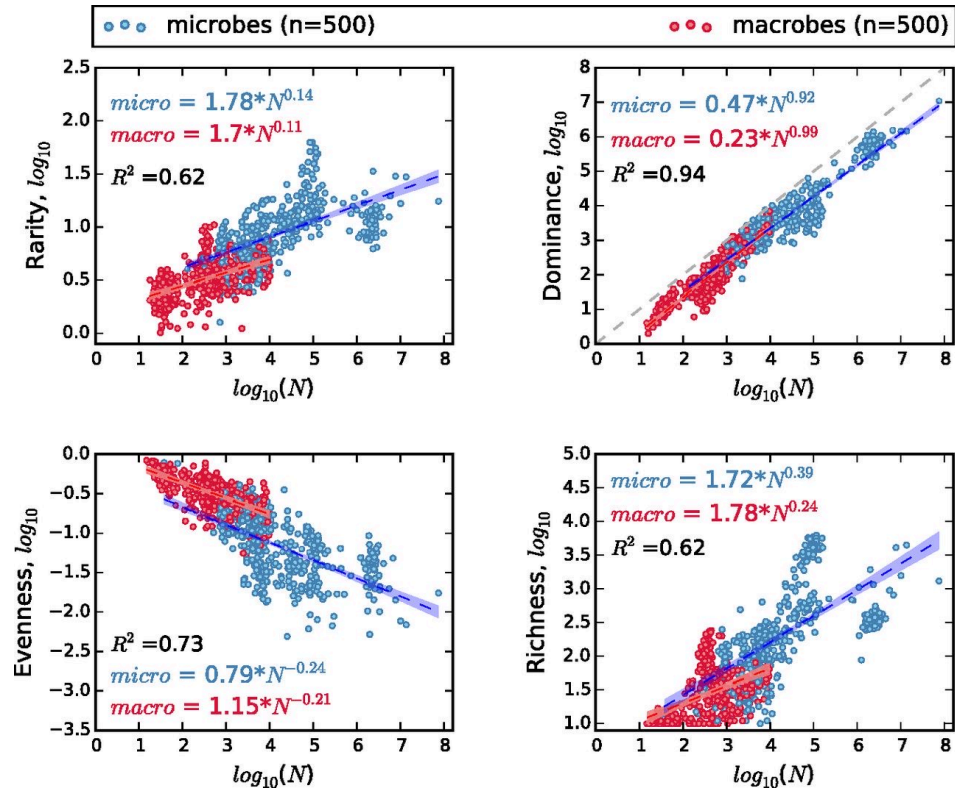
# Macroecological patterns



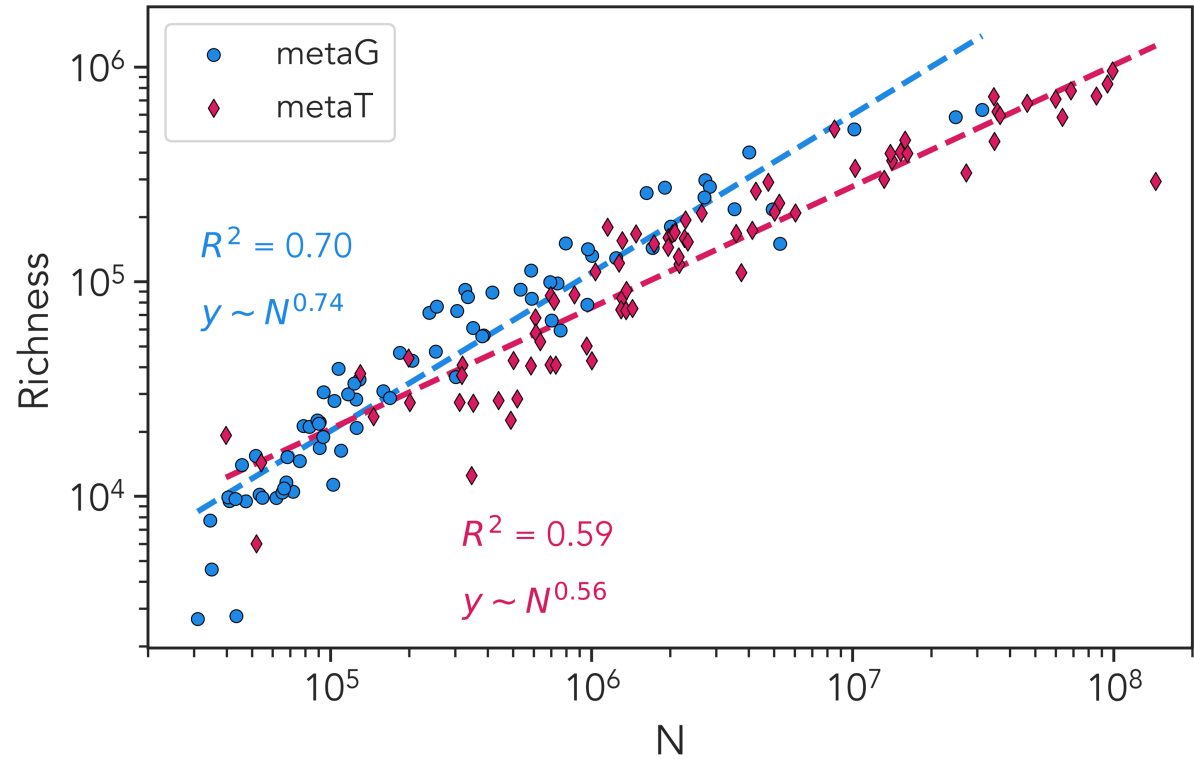
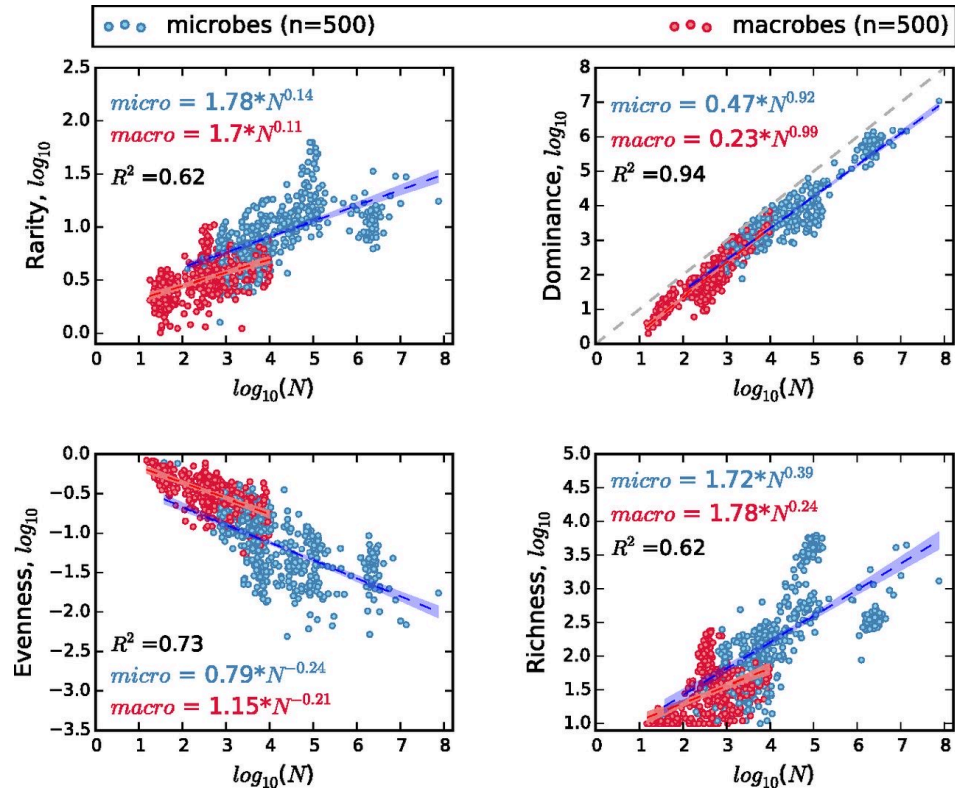
# Macroecological patterns



# Macroecological patterns



# Macroecological patterns





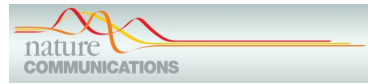
# Macroecological laws on the seascape

ARTICLE

<https://doi.org/10.1038/s41467-020-18529-y> OPEN

## Macroecological laws describe variation and diversity in microbial communities

Jacopo Grilli <sup>1,2</sup>



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## Scaling laws predict global microbial diversity

Kenneth J. Locey <sup>✉</sup> and Jay T. Lennon <sup>✉</sup> [Authors Info & Affiliations](#)

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