Delaunay-type compact equilibria in the liquid drop model

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Abstract

We deal with the *liquid drop model*, introduced by Gamow (1930) and Bohr-Wheeler (1939) in nuclear physics to describe the structure of atomic nuclei. The problem consists of finding a surface $\Sigma = \partial\Omega$ in $\mathbb{R}^3$ that is critical for the following energy of regions $\Omega \subset \mathbb{R}^3$:

$$\mathcal{E}(\Omega) = \mathrm{Per}(\Omega) + \frac{1}{2}\int_{\Omega\times\Omega} \frac{dxdy}{|x-y|}$$

under the volume constraint $|\Omega| = m$. The associated Euler-Lagrange equation is

$$H_\Sigma(x) + \int_\Omega \frac{dy}{|x-y|} = \lambda \quad \forall x \in \Sigma, \quad |\Omega| = m,$$

where λ is a constant Lagrange multiplier. Round spheres enclosing balls of volume m are always solutions. They are minimizers for sufficiently small m. Since the two terms in the energy compete, finding non-minimizing solutions can be challenging. We find a new class of solutions with large volumes, consisting of "pearl collars" with an axis located on a large circle, with a shape close to a Delaunay's unduloid surface with constant mean curvature. This is joint work with Monica Musso and Andrés Zúñiga.